Heat transfer within potatoes

Introduction

How long does it take to cook an egg, a roast, or a potato? The answer is generally a given duration, depending on the weight of the food. At the end of this experiment, we will see that this is typically incorrect. The following is an adaptation of [1]. A food item is cooked when its center reaches a particular temperature, T_c. The specific value of T_c depends on the details of the food item we are cooking. It characterizes when the food item is safe to eat or becomes edible (e.g., the temperature of pasteurization, the temperature of gelatinization of starches, or the temperature at which the proteins coagulate).

In this hands-on experiment, we will study the cooking process of potatoes by boiling them. Potatoes are cheap and easy to source. They are very familiar, and it is easy to tell whether they are cooked. When the potato reaches the temperature $T_c \approx 65°C$, the starch gelatinizes and becomes edible and translucent. Therefore, we can tell if they are cooked by cutting and looking at them (please do not eat them or use them as projectiles).

We model the potato as a uniform sphere of radius R_0 and thermal diffusivity κ. We cook it by boiling it. The boiling water cooking pan is a heat bath at a fixed temperature T_w. The temperature inside the potato satisfies the heat equation (temperature diffusion equation)

$$\partial_t T(\vec{x}, t) = \kappa \nabla^2 T(\vec{x}, t) \ . \tag{1}$$

Solving this equation is challenging (you can find a detailed derivation in [2] if you are interested, or ask us for some notes). The solution of equation (1) with appropriate boundary conditions can be found analytically

$$\frac{T(r,t) - T_w}{T_a - T_w} = 2 \sum_{n=1}^{\infty} (-1)^n \frac{R_0}{n\pi r} \sin\left(\frac{n\pi}{R_0} r\right) e^{-\kappa \left(\frac{n\pi}{R_0}\right)^2 t} \ . \tag{2}$$

Initially, the potato is at room temperature T_a. To find the cooking time t_c, we impose $T(0, t_c) = T_c$ and invert the function

$$\frac{T_c - T_w}{T_a - T_w} = 2 \sum_{n=1}^{\infty} (-1)^n e^{-\kappa \left(\frac{n\pi}{R_0}\right)^2 t_c} \approx 2 e^{-\kappa \frac{\pi^2}{R_0^2} t_c} \ , \quad \longrightarrow \quad t_c \approx \frac{R_0^2}{\kappa \pi^2} \log\left(\frac{1}{2} \frac{T_a - T_w}{T_c - T_w}\right) \ . \tag{3}$$

The cooking time is proportional to the radius square of the potato R_0^2, not its mass (or volume). If you assume the average thermal diffusivity of a potato $\kappa \approx 10^{-7} m^2 s^{-1}$ we can cook a potato from $T_a = 20°C$ with $T_w = 100°C$

$$t_c \approx 70s \left(\frac{R_0}{1cm}\right)^2 \ . \tag{4}$$

A potato with a $3cm$ radius cooks in about 10 minutes in this model, which is a reasonable result. Notice that this is a very rough estimate based on a simplified model.

If we wait a shorter time t, only a shell of thickness x will be cooked. The thickness of the shell is proportional to the square root of the cooking time.

> Your goal in the next two and a half hours is to verify the relation between x and t experimentally
>
> $$x \propto \sqrt{t} \ .$$

[1] P. Barham, *The Science of Cooking*, Springer-Verlag, Berlin Heidelberg, 2001.

[2] L.R. Wang, Y.J. Jin, and J.J. Wang, *A simple and low-cost experimental method to determine the thermal diffusivity of various types of foods* Am. J. Phys. 90, 568–572 (2022)

Materials

- 6 potatoes
- Water
- Cooking pan
- Knife
- Stove
- Caliper

Procedure

- Draw the following table

Duration (t in min)	x_{min}	x_{max}
1.0		
2.0		
3.5		
5.5		
7.5		
10.0		

  This is the minimum amount of measurements we require you to make. You are free to make more of them if you want.

- Bring water to a boil.
- Introduce the potatoes inside the boiling water (we suggest boiling one potato at a time for every corresponding duration indicated in the table).
- Remove the potato and cut it in half.
- Use the caliper to measure the width of the translucent potato ring at different points (at least three). Write down the min and max values on the table.
- Plot the average distance (with error bars) as a function of using spreadsheet software.

Questions

- Does the variation of x with t follow the expected trend?
- Can you fit a square root function to the data points?
- Discuss the possible deviation between the expected and the observed trend.
- What changes if we peel the potato? Why?
- How can we improve our model? What important effects are we neglecting?