Mesuring the focal length of a thin lens

Pietro Dona

December 28, 2023

1 Context

I supervised first-year PeiP students in the final laboratory projects of the mechanics and geometrical optics course at Aix-Marseille University in the winter of 2023. The Integrated Preparatory Cycle (PeiP: Parcours des Ecoles d'Ingénieurs Polytech) aims to prepare students for the Polytech engineering cycle. I was supervising small groups of 15 students doing a geometrical optics project. The experiment's most significant part required precise measurements of the focal length of many lenses. I prepared this document before the lab sessions to prepare for questions from the students. I integrated it and published it after the course during the Christmas break.

2 Focometry

Focometry in geometric optics determines a lens's power P and focal length f. The power of a lens, measured in diopters, indicates its ability to converge or diverge light. Focal length is the distance from the lens at which parallel rays converge or appear to diverge. The two are one the inverse of the other $P = 1/f$. We propose and describe three independent methods to measure the focal length of a convex lens. We will also measure the focal length of a divergent lens by coupling it with a convergent one. The first method we explore is using the thin lens equation and multiple direct measurements of object/screen distances to compute the focal length. The second method uses autocollimation, placing an object in a focal point and reflecting the image back on it with a mirror. The last method exploits the definition of the focal length as the distance from the lens and the point where parallel rays converge.

3 Thin lens equation

The thin lens equation

$$\frac{1}{s} + \frac{1}{t} = \frac{1}{f} \tag{1}$$

relates the distance between the lens and the object s, the distance between the lens and the screen t, and the focal length f. Knowledge of s and t gives us access to f. We report the general light ray tracing scheme in Figure 1.

Materials. We perform this measurement using a graduated optical bench, a lamp fixed at one end of the bench, a semi-transparent object we use as a source, a lens mounted on support we can move along the bench and a large white screen that we can also move along the bench.

Protocol. We mount the object right in front of the lamp. We keep the lamp, the object, the lens, and the screen aligned. We choose a distance s and place the lens at that distance from the object. The uncertainty on s is given by the resolution of the optical bench of $1\,mm$. We move the screen until we find a clearly formed image on it. In general, we will find an interval of distances t compatible with a focused image. The interval t_{min} and t_{max} (the minimal and maximal values) represent the uncertainty on the measurement of

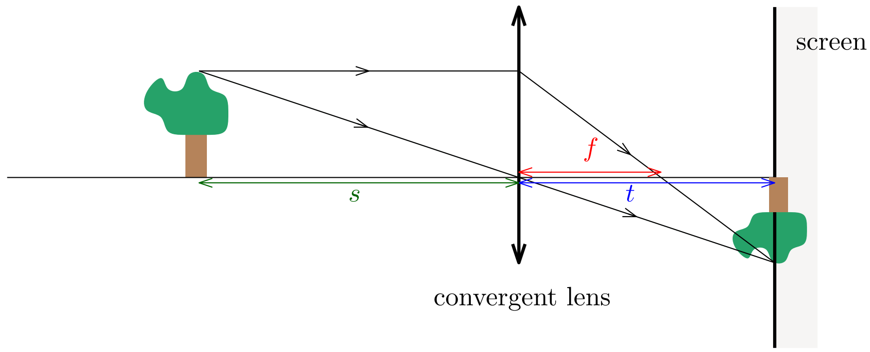


Figure 1: Generic schema of image forming on a screen with a convergent lens. We find the image using the two fundamental properties of a thin lens. Light rays passing through the optical center of the lens are not deflected. Light rays orthogonal to the lens are deflected and intersect in the focal point. the intersection of the extremal rays determines the position of the image we capture with a screen.

t. We repeat the process for various values of s, finding the corresponding t. We plot $\frac{1}{t}$ as a function of $\frac{1}{s}$ and perform a linear fit to find $\frac{1}{f}$ (and f) as the intercept of the straight line.

Measurements. We work with a lens ℓ_{15cm} with a nominal focal distance of $15\,cm$. We take $20\,cm$ as our minimal value of s, a simple round number slightly larger than the nominal focal distance. In this way, we can comfortably form the image (expected at around $60\,cm$) on the screen within the length of the optical guide (more or less $1\,m$). Similarly, we fix the maximal value of s at $60\,cm$ leveraging the formal symmetry of the problem under the exchange of screen and object. With the final goal in mind, we choose other 5 intermediate distances s so their inverses are equidistant. We report the result of our measures in the Table 1. We plot the data in Figure 2. We perform two kinds of linear fits with

$s \pm 0.1(cm)$	$t_{min}(cm)$	$t_{max}(cm)$
60.0	19.4	19.9
45.0	21.9	22.4
36.0	24.8	25.5
30.0	29.0	29.5
25.7	34.7	35.1
22.5	43.0	43.9
20.0	56.7	57.3

Table 1: Raw measures of the distance of the object (s) and screen (t_{min} and t_{max}). All the measurements are in cm. The interval for the screen distances is compatible with a well-focused image.

$$y = mx + b \; . \tag{2}$$

First, we draw a straight line passing through the maximum of the first point and the minimum of the last point and a straight line passing through the minimum of the first point and the maximum of the last point. We find

$$m = (-0.97, -1.02) \;, \qquad b = (0.066, 0.068)\,cm^{-1} \; . \tag{3}$$

The number of significant digits is slightly incorrect. We will keep them anyway for illustrative reasons. Second, we perform a least square fit with a linear function. We find

$$m = -1.0 \;, \qquad b = 0.067\,cm^{-1} \; . \tag{4}$$

In both quantities, the uncertainty is smaller than the reported significant digits. From both, we can conclude that $f_{15cm} = (14.6, 15.0)cm$.

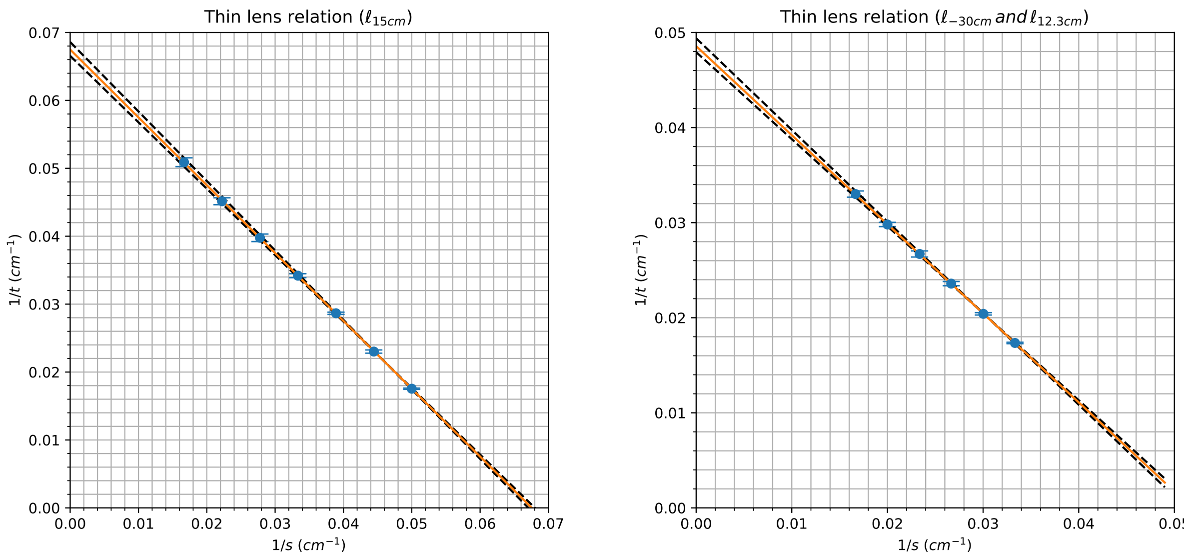


Figure 2: Plot of the data in Table 1. We plot the inverse of the distances of the object and the screen from the lens. We fit the data with straight lines. The two dashed lines are extremal lines compatible with the data points. the orange line is the best fit done with the least square method. Notice that the formula is symmetric under the exchange of s and t. Consequently, the plot is symmetric by exchange of the x and y-axis. *Left panel.* Plot for the convergent lens f_{15cm}. *Right panel.* Plot for the system of one convergent and one divergent lens f_{15cm}.

4 Thin lens equation and divergent lens

To determine the focal length of a composition of two lenses, we can apply the thin lens equation and use the schema in Figure 3. We find the exact formula

$$\frac{1}{s} + \frac{1}{e - \frac{1}{\frac{1}{f_2} - \frac{1}{t}}} = \frac{1}{f_1} \ . \tag{5}$$

If the distance between the two lenses is very small $e \ll f_2, t$, we can ignore e and obtain the well-known formula

$$\frac{1}{s} + \frac{1}{t} = \frac{1}{f_1} + \frac{1}{f_2} \ . \tag{6}$$

Alternatively, if e/f_2 is small but not negligible, we can expand in series for small e/f_2 and find

$$\frac{1}{s} + \frac{1}{t}\left(1 + 2\frac{e}{f_2}\right) = \frac{1}{f_1} + \frac{1}{f_2}\left(1 + \frac{e}{f_2}\right) \ . \tag{7}$$

Note that we are also neglecting a e/t^2 term. We use the formula (7) to measure a divergent lens's focal length. We follow the same protocol we described for the convergent lens. We pair a divergent lens with a nominal focal length of $f_{-30cm} = -30cm$ with a convergent lens with a focal length $f_{12.5cm} = 12.3 \pm 0.2cm$ we measured accurately. The distance between the center of the two lenses is estimated to be $e = 0.6cm$. We report the raw measurements in Table 2. We plot the data in Figure 2 together with a linear fit of the angular coefficient and the intercept of

$$m = (-0.96, -0.9) \qquad b = (0.048, 0.049)\, cm^{-1} \ . \tag{8}$$

Comparing with the formula (7) we find a huge interval for $f_{-30cm} = (-15, -30)\, cm$ from the angular coefficient. We can refine it using $f_{12.5cm} = 12.3 \pm 0.2cm$ and the value of the intercept. We find an interval

$s \pm 0.1(cm)$	$t_{min}(cm)$	$t_{max}(cm)$
60.0	30.0	30.6
50.0	33.3	33.8
42.8	37.0	37.9
37.5	42.0	42.8
33.3	48.7	49.3
30.0	57.4	57.9

Table 2: Raw measures of the distance of the object (s) and screen (t_{min} and t_{max}) for the composition of divergent and convergent lenses. All the measurements are in cm. The interval for the screen distances is compatible with a well-focused image.

for

$$f_{-30cm} = (-28.2, -32.1)\, cm \ . \tag{9}$$

We find a satisfactory agreement with our expectations. Notice that by switching the lenses around, we get a straight line with angular coefficient $m < -1$ and a simpler formula for the focal distance.

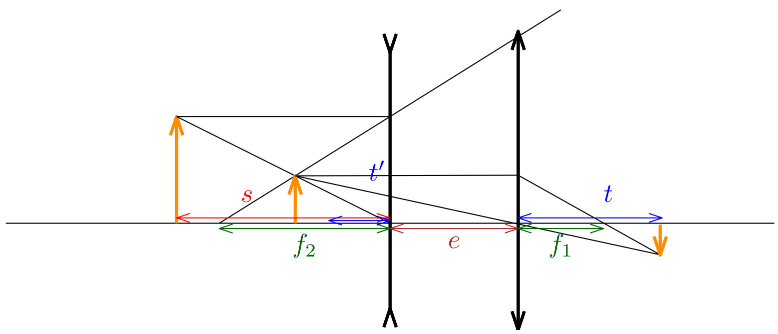


Figure 3: Generic schema of image forming on a screen with an optical system of two lenses, one divergent and one convergent. The virtual image of the divergent lens is focused by the convergent one.

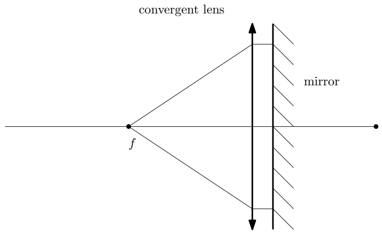
5 Autocollimation

If we place a point object in the focal point of a lens, the image forms at infinity. On the other hand, if we put an object at infinity, the image is formed in the focal plane. In equation (1), put s or t to infinity. To measure the focal distance of a lens, we can take advantage of this property by placing a reflecting mirror right after the lens to send the image back to the object. If (by moving the lens) we put the object in the focal point of the lens, its image will form on top of the object itself (see Figure 4a). We put the lens at a certain distance s and keep a mirror attached. We cover the upper half of the object with a white plastic card (e.g., the metro card taped to a sheet of paper). We slowly change the position of the lens until we see the image of the lower half of the subject formed on the covered half. At this point, we measure the distance between the lens and the object and use it as an estimate of the lens's focal length.

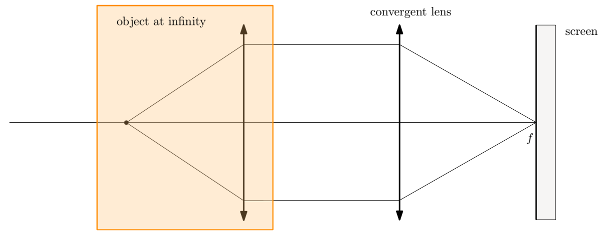
We find

$$f_{15cm} = (14.6, 15.2)\, cm \ . \tag{10}$$

The intervals of measurements compatible with the focal length correspond to the minimal and maximal distances for which we can say that the image is well-formed. A group of students also performed the autocollimation with divergent lenses. This is impossible, and the image they see is not due to autocollimation. In fact, we could form the image even without the mirror. The image is an effect of the reflection of the surface of the lens. Since we have no information on the shape of the lens relating the curvature of the surface to the focal length, it is dangerous.



(a) Schema for autocollimation. When the object is placed at the focal point the reflected image forms on the object itself.



(b) Schema for the object at infinity. When the object is placed at infinity (orange box), the image forms on the focal point of the lens.

6 Object at infinity

We use a pre-build system of a convergent lens plus an object placed precisely in its focal point (see Figure 4b). The light rays coming from this "object at infinity" are all parallel as the object was placed at infinity. In this case, the source is pre-build, and we assume its characteristics. As the light rays are all parallel, the distance between the *object at infinity* and the lens we want to study is irrelevant. As the object is at infinity, the image will form at the focal point of the lens in the exam. We move the screen until the image is well-formed and estimate the lens's focal length, measuring its distance from the screen.

We measure the focal length of the same three convergent lenses as the autocollimation method and find

$$f_{15cm} = (14.3, 15.4)\, cm \ . \tag{11}$$

The intervals of measurements compatible with the focal length correspond to the minimal and maximal distances for which we can say that the image is well-formed. We observe a generally higher uncertainty than the previous method.