Measuring the size of an object and verification of a physical model

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December 24, 2023

1 Objective

The objective of the experiment is to verify the relation between the distance and the angular diameter of an object.

2 Setup

The relation between the angular diameter $\tan\alpha$, the size of the object H, and the distance D we want to verify is

$$\tan\alpha = \frac{H}{D}\ . \tag{1}$$

We pay attention to keeping the observer at the same height as the object's base. Using a ruler, we measure the angular diameter $\tan\alpha$ of the objects at various distances. We fix the distance D from the observer to the object. We place a ruler at a distance d and read the object's apparent size h on the ruler looking from the observer spot. We compute the angular diameter using the relation

$$\tan\alpha = \frac{h}{d}\ . \tag{2}$$

We report a schematic representation of the setup in Figure 1.

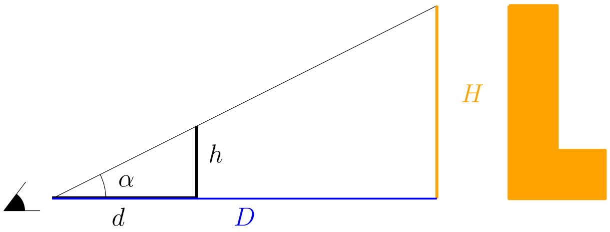

Figure 1: Schema of the experiment.

3 Measurements

We set the distance between the object and observer to $D = 2m$. We set a ruler at a distance of $d = 50cm$ and measured the object's apparent size h. We move the ruler at a distance $d = 55cm$ and $d = 60cm$, and we measure h each time. We repeat the measurements twice. Using different values of d helps reduce measurement bias due to the difficulty of aligning the object. We report all the measurements in the Table 1. For every measurement, we compute the angular diameters using the relation 2.

$d(cm)$	$h_{D=2.0m}(cm)$	$h_{D=1.8m}(cm)$	$h_{D=1.6m}(cm)$	$h_{D=1.4m}(cm)$	$h_{D=1.2m}(cm)$	$h_{D=1.0m}(cm)$
50.0	5.6	5.9	7.2	7.8	9.5	11.3
50.0	5.9	6.3	7.4	7.4	8.7	10.8
55.0	6.4	6.9	7.8	9.2	10.0	12.6
55.0	6.2	7.0	8.0	9.4	10.3	12.8
60.0	6.7	7.3	8.7	9.5	10.9	13.0
60.0	6.2	6.8	7.8	9.3	11.2	12.9

Table 1: Raw measurements of the apparent size.

$D = 2.0m$	$D = 1.8m$	$D = 1.6m$	$D = 1.4m$	$D = 1.2m$	$D = 1.0m$
0.11	0.12	0.14	0.16	0.19	0.23
0.12	0.13	0.15	0.15	0.17	0.22
0.12	0.13	0.14	0.17	0.18	0.23
0.11	0.13	0.15	0.17	0.19	0.23
0.11	0.12	0.14	0.16	0.18	0.22
0.10	0.11	0.13	0.15	0.19	0.21

Table 2: Derivation of the angular diameter for various distance values.

4 Data Analysis

For each distance, we compute the average, the min, and the maximum angular diameters. The min and the max values can be used to estimate an uncertainty on the average. We can plot the data $y = \tan\alpha$ as a function of $x = 1/D$ to recover an approximately linear behavior. See Figure 2. We compare the data using a linear model $y = Hx$ with no intercept. The absence of intercept can be justified by the physical assumption that for the infinite distance, the angular diameter vanishes. We extract the largest and smallest angular coefficients, computing the increments $H = \Delta y/\Delta x$ with respect to the origin. We find $21.6cm \leq H \leq 22.4cm$.

$D(cm)$	$\tan(\alpha)$
200	$0.11^{+0.01}_{-0.01}$
180	$0.12^{+0.01}_{-0.00}$
160	$0.13^{+0.01}_{-0.01}$
140	$0.15^{+0.02}_{-0.01}$
120	$0.18^{+0.01}_{-0.01}$
100	$0.22^{+0.01}_{-0.02}$

5 Conclusion

We verified that the angular diameter of an object is inversely proportional to the distance from the object (2). We estimated the proportionality coefficient to be within the range $21.6cm \leq H \leq 22.4cm$, which is compatible with the real size of the object of $22cm$

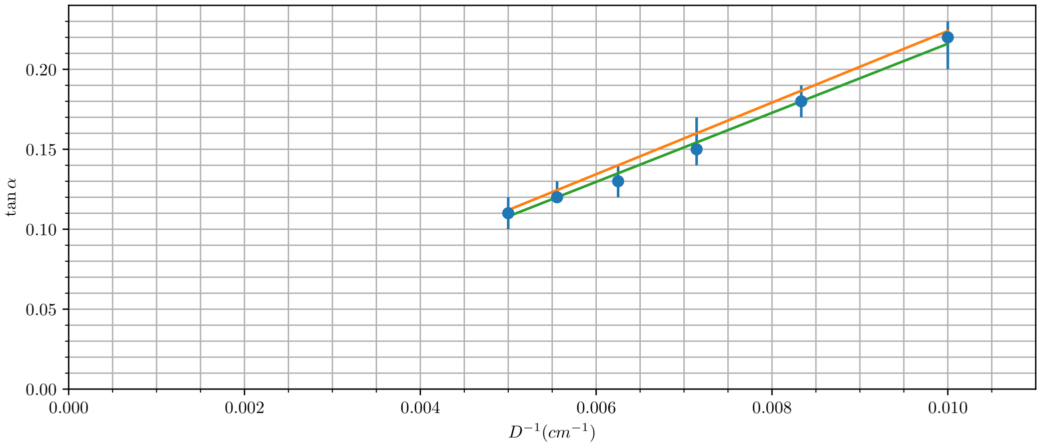

Figure 2: Plot of $\tan\alpha$ as a function of $D^{-1}(cm)$. We also report the maximal and minimal linear fits.