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Can we compute spin foam amplitudes on a computer?

University of New Brunswick Gravity Group Meetings - 6th May 2022

Collaborators and Bibliography

Simone Speziale (*motivator*)

François Collet (*first attempt*)

Giorgio Sarno (*pioneer*)

Francesco Gozzini (*master coder*)

Pietropaolo Frisoni (*maintainer*)

[S. Speziale - 2017]

[P.D. M. Fanizza, G. Sarno S. Speziale – 2018]

sl2cfoam [P.D. G. Sarno – 2018]

[P.D. M. Fanizza, G. Sarno S. Speziale – 2019]

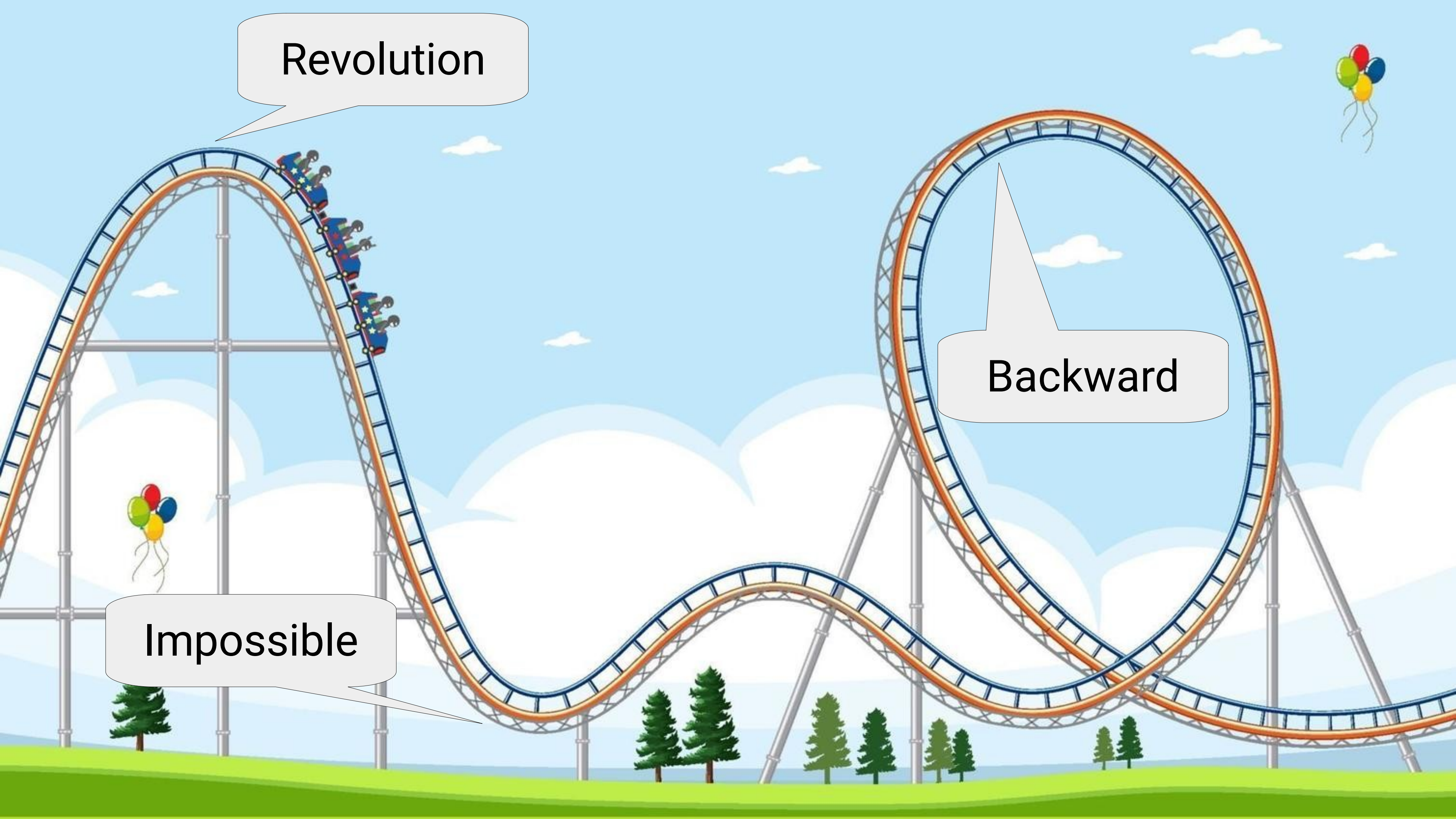
[P.D. G. Sarno F. Gozzini – 2020]

[P.D. G. Sarno F. Gozzini – 2021]

sl2cfoam-next [F. Gozzini – 2021]

[P. Frisoni, F. Gozzini, F. Vidotto – 2022]

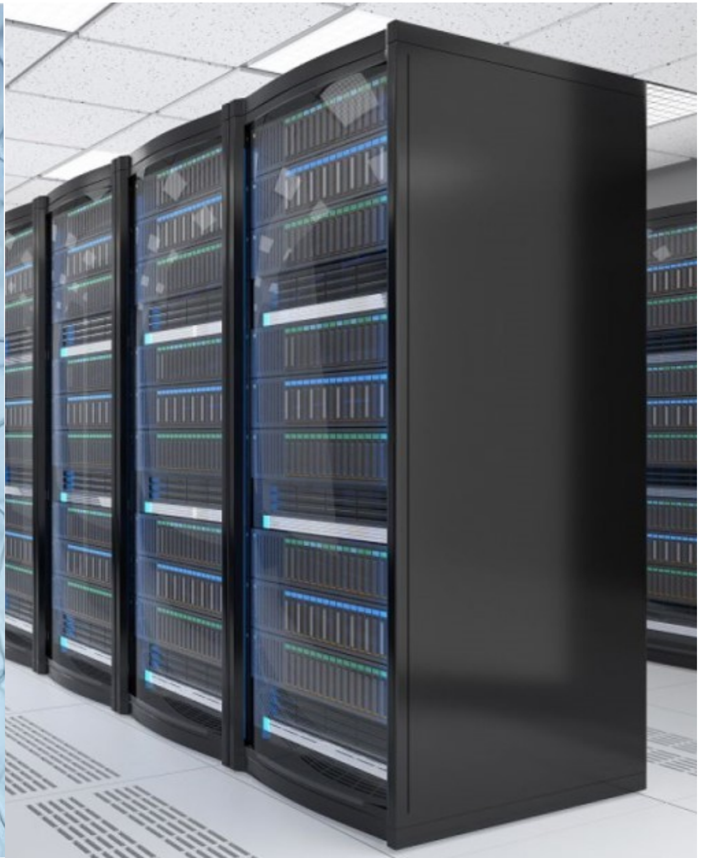
[P.D., P. Frisoni – 2022]


Revolution
Backward
Impossible

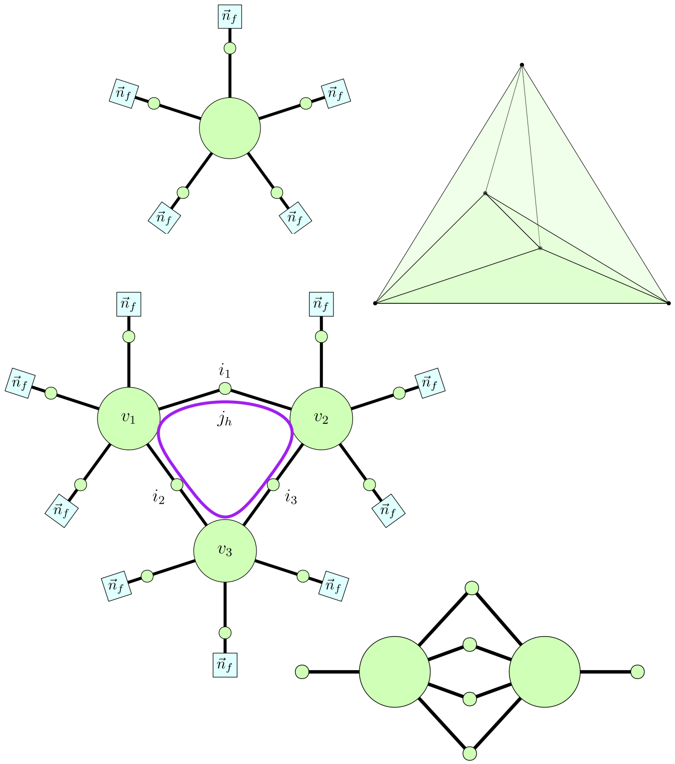
Plan for the talk

Spin foams?

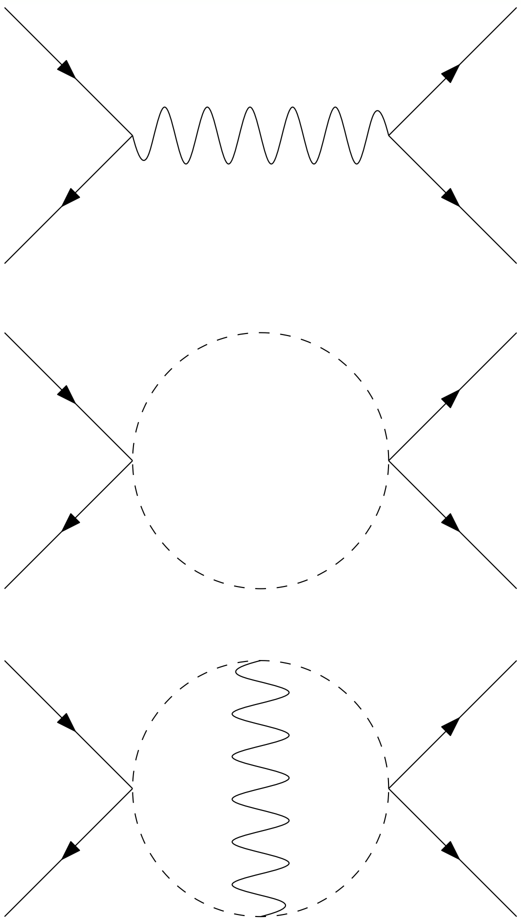
Numerics?



To do what?



Transition amplitudes in QFT



Boundary States = Free particles

Asymptotic states

Transition Amplitudes = Path Integral

Interaction vertices

Propagators

Calculation = Feynman diagrams

Truncation (# loops)

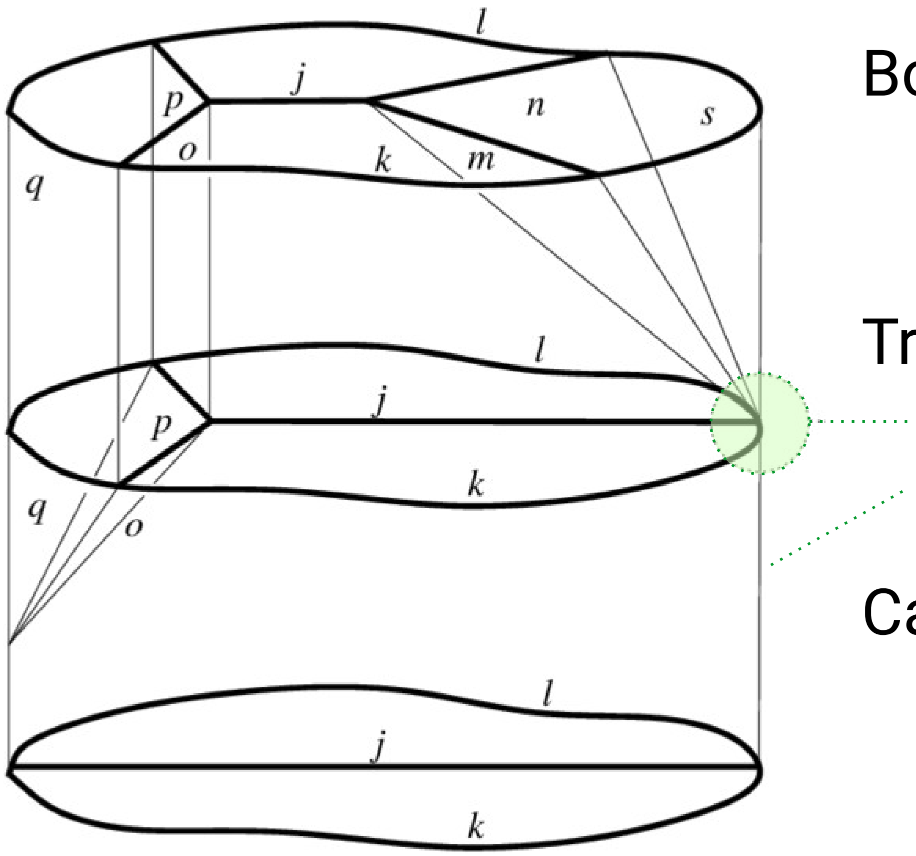
Contract vertices with propagators

Integrate over “bulk” variables

Transition amplitudes in LQG

[A. Perez - 2013]

[C. Rovelli and F. Vidotto - 2014]



Boundary States = Kinematical LQG state

Quantum space states (spin networks)

Transition Amplitudes = EPRL theory

Interaction vertex

Propagators

Calculation = Spin foam diagrams

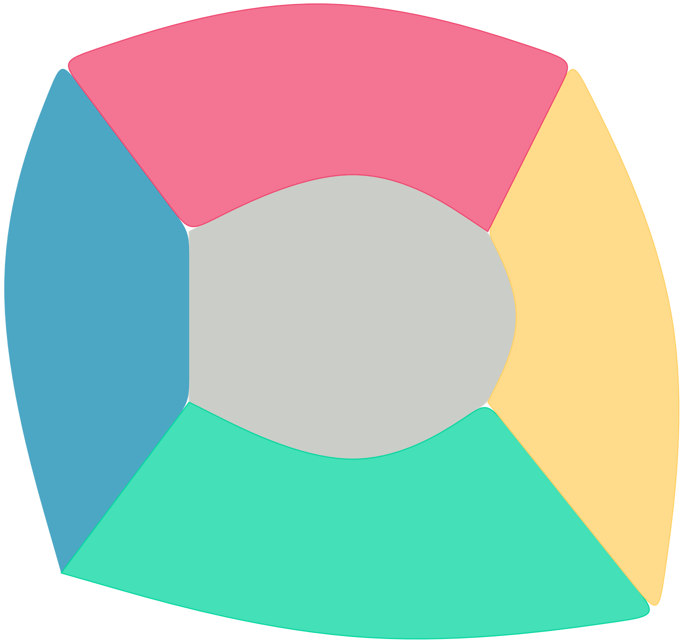
[Oriti - 2013]

Truncation (# vertices)

Contract vertices with propagators

Sum over “bulk” quantum numbers

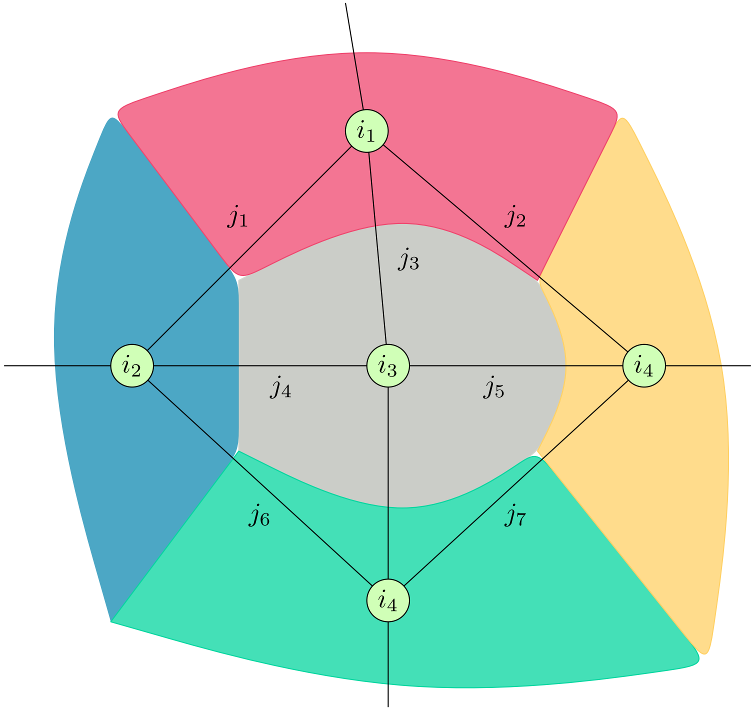
Canonical LQG in one slide (boundary states)



Kinematical Hilbert space

- Granular states of quantum space (reference frames)
- Geometric operators with discrete spectrum
- Finite truncation of gravitational d.o.f

Canonical LQG in one slide (boundary states)



Kinematical Hilbert space

Granular states of quantum space (reference frames)

Geometric operators with discrete spectrum

Finite truncation of gravitational d.o.f

Spin network basis

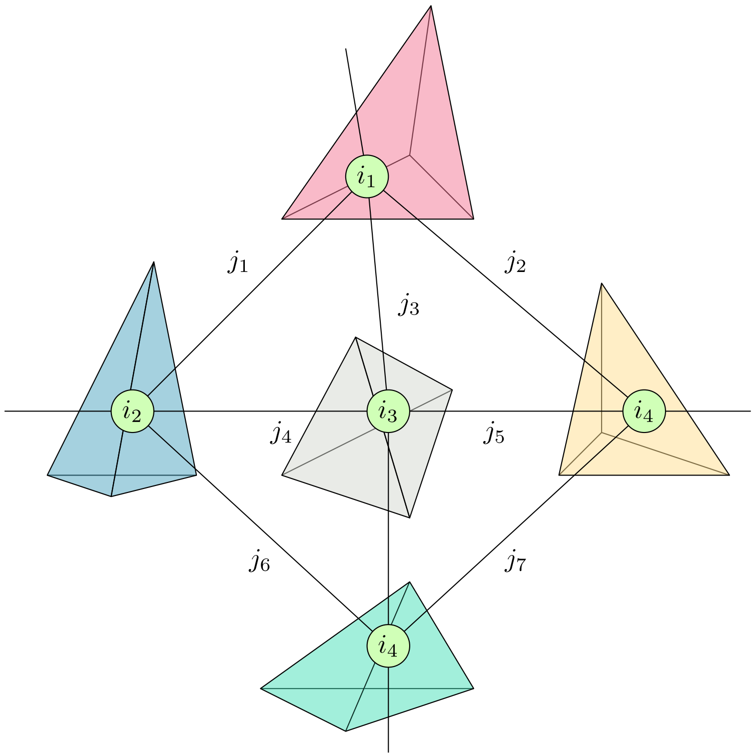
Graph = Adjacency

Spins on links (j_f)= Areas

Intertwiners on nodes (i_e) = Volumes & shapes

Parallel transport (Ashtekar-Barbero holonomy)

Canonical LQG in one slide (boundary states)



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Parallel transport (Ashtekar-Barbero holonomy)

Abstract graph, SU(2) invariance

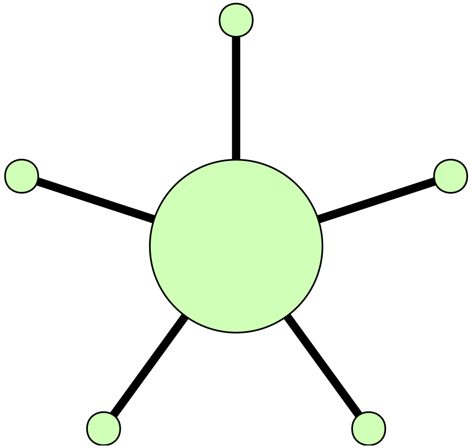
Quantum polyhedra (*tetrahedra*) [E. Bianchi, P.D. S. Speziale – 2011]

Twisted geometries [L. Freidel, E. Livine, S. Speziale – 2012]

The (simplicial) spin foam theory

(simplicial) Interaction vertex

Interaction between 5 quantum tetrahedra (edges)

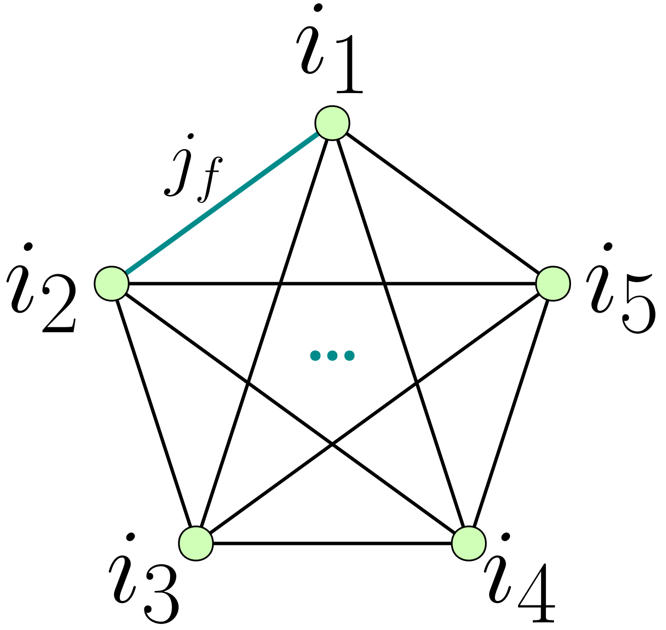


The (simplicial) spinfoam amplitude

(simplicial) Interaction vertex

- Interaction between 5 quantum tetrahedra (edges)
- Adjacency of the boundary a 4-simplex (faces)
- Quantum number (momentum space)
- Vertex amplitude (details will come later)

$$A_v(j_f,\ i_e)$$



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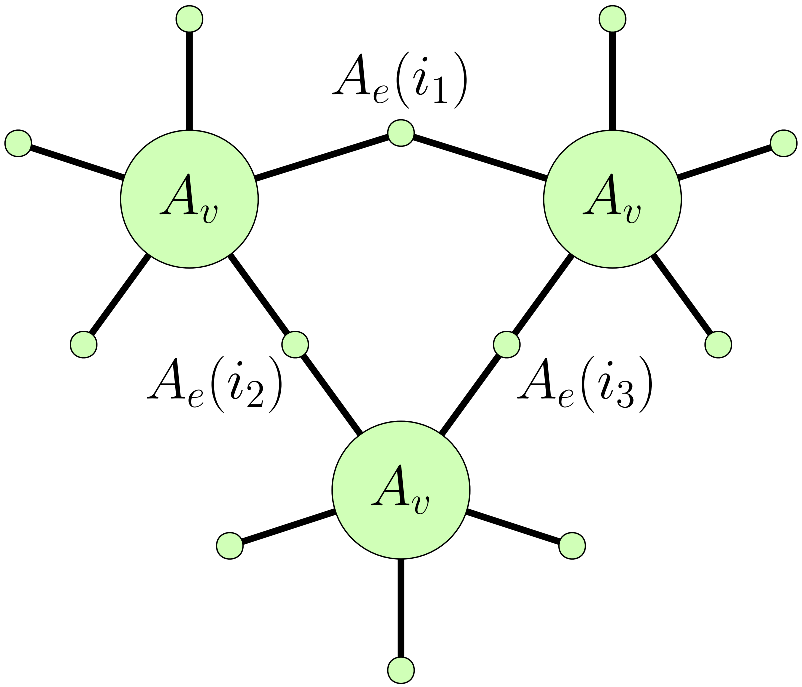

Propagator

Quantum tetrahedra shared between vertices

$$A_e(i_e) = (2i_e + 1)\ \delta_{i_e i_{e'}}$$

The (simplicial) spinfoam amplitude

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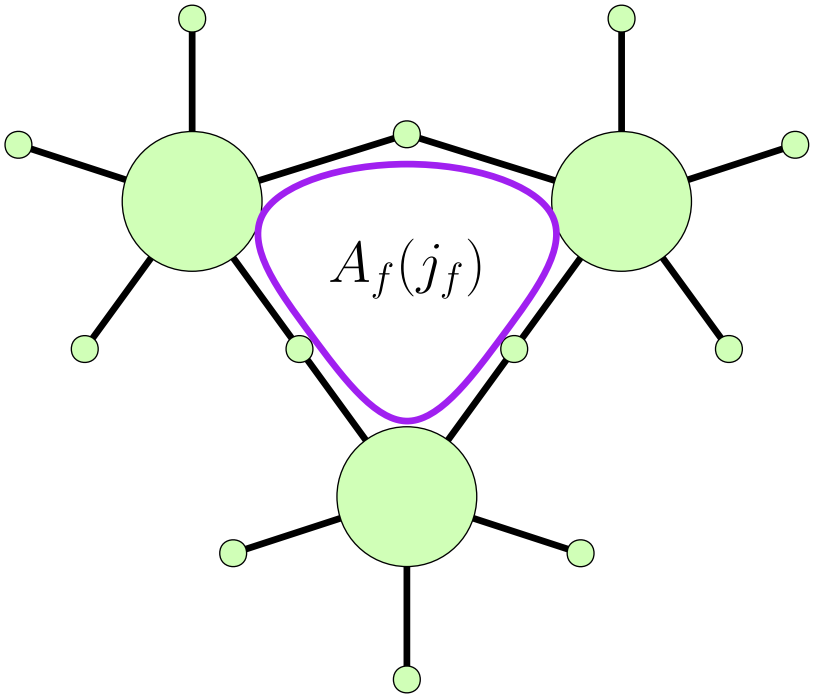


Propagator

Quantum tetrahedra shared between vertices

$$A_e(i_e) = (2i_e + 1)\ \delta_{i_e i_{e'}}$$

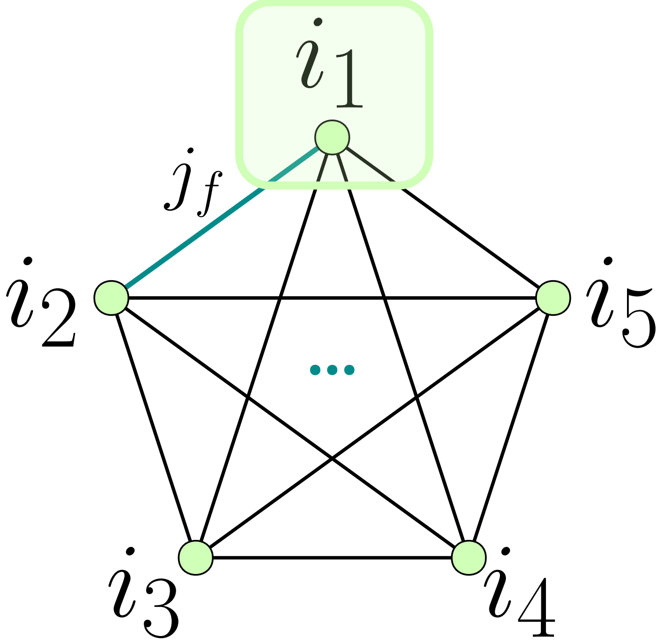
Transition amplitude

$$A_\Delta = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f,\ i_e)$$

$$A_f(j_f) = 2j_f + 1$$

The EPRL vertex amplitude

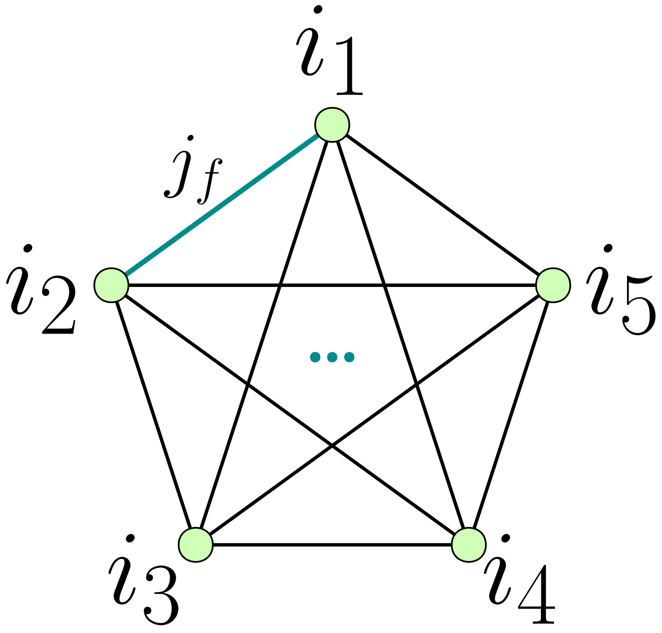
$$A_v(j_f,\ i_e) = \int \prod_{e\in v} \mathrm{d}g_e\, \delta(g_1) \sum_m \prod_{f\in v} D^{\gamma j_f, j_f}_{j_f m_{f_t}, j_f m_{f_s}}(g_t^{-1} g_s) \prod_{e\ni f} \begin{pmatrix} j_f \\ m_p \end{pmatrix}^{(i_e)}$$



One $\mathrm{SU}(2)$ invariant tensor per quantum tetrahedron
(*Wigner (4jm) symbol*)

The EPRL vertex amplitude

$$A_v\left(j_f,\ i_e\right)=\int\prod_{e\in v}\mathrm{d}g_e\,\delta(g_1)\sum_m\prod_{f\in v}D^{\gamma j_f, j_f}_{j_f m_{f_t}, j_f m_{f_s}}(g_t^{-1}g_s)\prod_{e\ni f}\begin{pmatrix} j_f \\ m_p \end{pmatrix}^{(i_e)}$$



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Lorentzian, $SL(2,\mathbb{C})$ gauge group (*parallel transport*)

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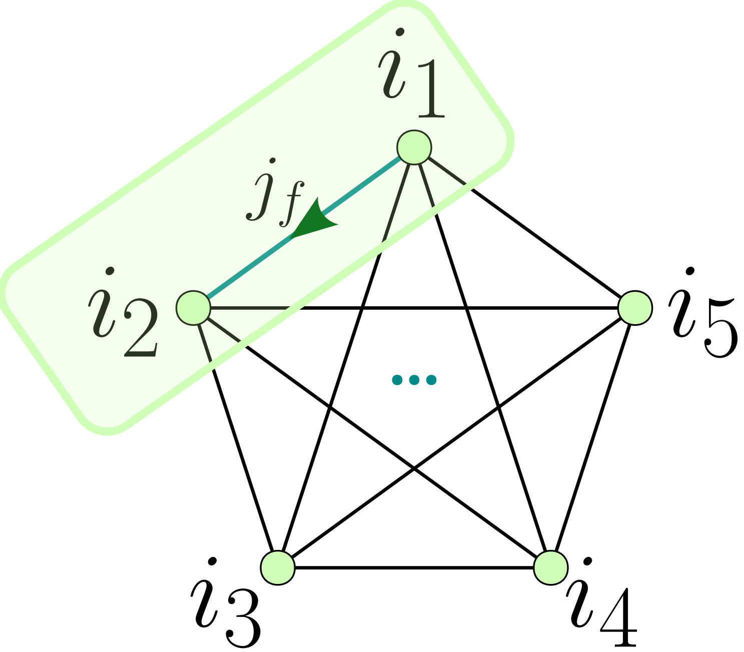

One $\mathrm{SU}(2)$ invariant tensor per quantum tetrahedron (*Wigner (4jm) symbol*)

Lorentzian, $SL(2,\mathbb{C})$ gauge group (*parallel transport*)

Matrix element in a *gamma simple* unitary irreducible representation of the Lorentz group (*infinite dimensional*) obtained by the face spin.

(γ Immirzi parameter)

Booster decomposition of the vertex amplitude

[S. Speziale - 2017]

$$A_v(j_f, i_e) = \sum_{l_f = j_f}^{\infty} \sum_{k_e} \left(\prod_e d_{k_e} B_4(j_f, l_f, i_e, k_e) \right) \{15j\}(l_f, k_e)$$

$$B_4(l_f, j_f, i_e, k_e) \equiv \sum_{m_f} \begin{pmatrix} l_f \\ m_f \end{pmatrix}^{(i_e)} \left(\int_0^\infty d\mu(r) \prod_f d^{(\gamma j_f, j_f)}_{l_f j_f m_f}(r) \right) \begin{pmatrix} j_f \\ m_f \end{pmatrix}^{(k_e)}$$

Linear superposition of SU(2) invariants weighted by Booster functions

- Six sum over virtual spins (*representation* property)
- Wigner {15j} symbols (SU(2) invariant)
- One Booster function on each edge (minus one - *Cartan decomposition*)
- Repeatable and parallelizable tasks
- Numerical implementation (*sl2cfoam and sl2cfoam-next*)

sl2cfoam-next

[F. Gozzini – 2021]

Advantages

- Open source
- Written in C
- Modular
- Scalable
- Optimized for HPC
- User friendly (Julia)

Disadvantages

- Resource demanding
- Unavoidable approximation

$$A_v(j_f, i_e) = \underbrace{\sum_{l_f=j_f}^{\infty} \sum_{k_e}}_{3} \left(\prod_e d_{k_e} \underbrace{B_4(j_f, l_f, i_e, k_e)}_{2} \right) \underbrace{\{15j\}(l_f, k_e)}_{1}$$

SU(2) invariants & Booster functions

SU(2) invariants are well studied

- Important in other physics and science fields (spectroscopy, nuclear physics, or chemistry)
- Wigner {15j} symbols as a sum of {6j} symbols (of the first kind)
- Do not reinvent the wheel (*WIGXJPF and FASTWIGXJ*) [H. T. Johansson and C. Forssén - 2015]

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Booster functions are more complicated

$$B_4(l_f, j_f, i_e, k_e) \equiv \sum_{m_f} \begin{pmatrix} l_f \\ m_f \end{pmatrix}^{(i_e)} \left(\int_0^\infty d\mu(r) \prod_f d^{(\gamma j_f, j_f)}_{l_f j_f m_f}(r) \right) \begin{pmatrix} j_f \\ m_f \end{pmatrix}^{(k_e)}$$

- One unbounded dimensional integral? Not really.
- Highly oscillating (arbitrary precision routines)
- Adaptive Gauss–Kronrod quadrature method (finer sampling where needed)

Summing everything up

$$A_v(j_f, i_e) = \sum_{l_f = j_f}^{\infty} \sum_{k_e} \left(\prod_e d_{k_e} B_4(j_f, l_f, i_e, k_e) \right) \{15j\}(l_f, k_e)$$

Unbounded sums

A relic of the non compactness of the group

Numerics requires a (*uniform*) approximation

$$\sum_{l=j_f}^{\infty} \to \sum_{l=j_f}^{j_f + \Delta l}$$

We know it is a good approximation (amplitude is finite)

We do not know how good (weak control on the error)

Convergence acceleration technique (*Aitken delta-squared*)

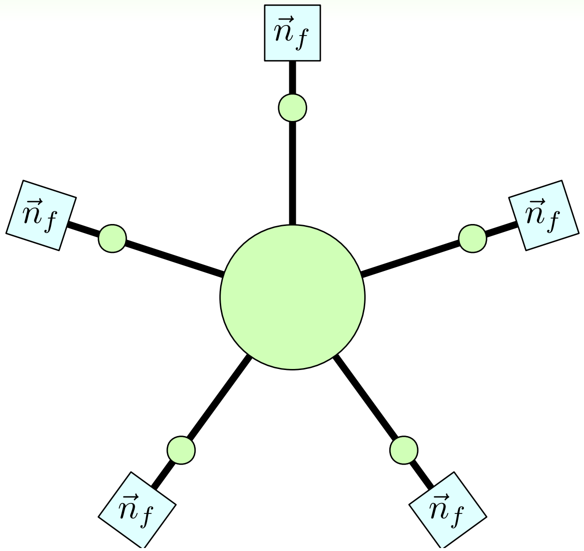
Applications - Quickstart

[P.D., P. Frisoni – 2022]

```
using SL2Cfoam
Immirzi = 1.2
data_folder = "path_data_folder"
configuration = SL2Cfoam.Config(VerbosityOff, VeryHighAccuracy, 100, 0)
SL2Cfoam.cinit(data_folder, Immirzi, configuration)
boundary_spins = ones(10)
Dl = 15
Av = vertex_compute(boundary_spins, Dl)
```

Applications – Single vertex asymptotic

$$A_v(j_f, \vec{n}_f) = \sum_{i_e} A_v(j_f, i_e) \prod_e c_{i_e}(\vec{n}_f)$$



Semiclassical limit

Uniform rescaling of the boundary spins $j_f \to \lambda j_f$

Boundary data representing Lorentzian 4-simplex (Livine-Speziale coherent states)

Exponentials of the Regge action (action for discrete GR)

[Barret & co - 2011]

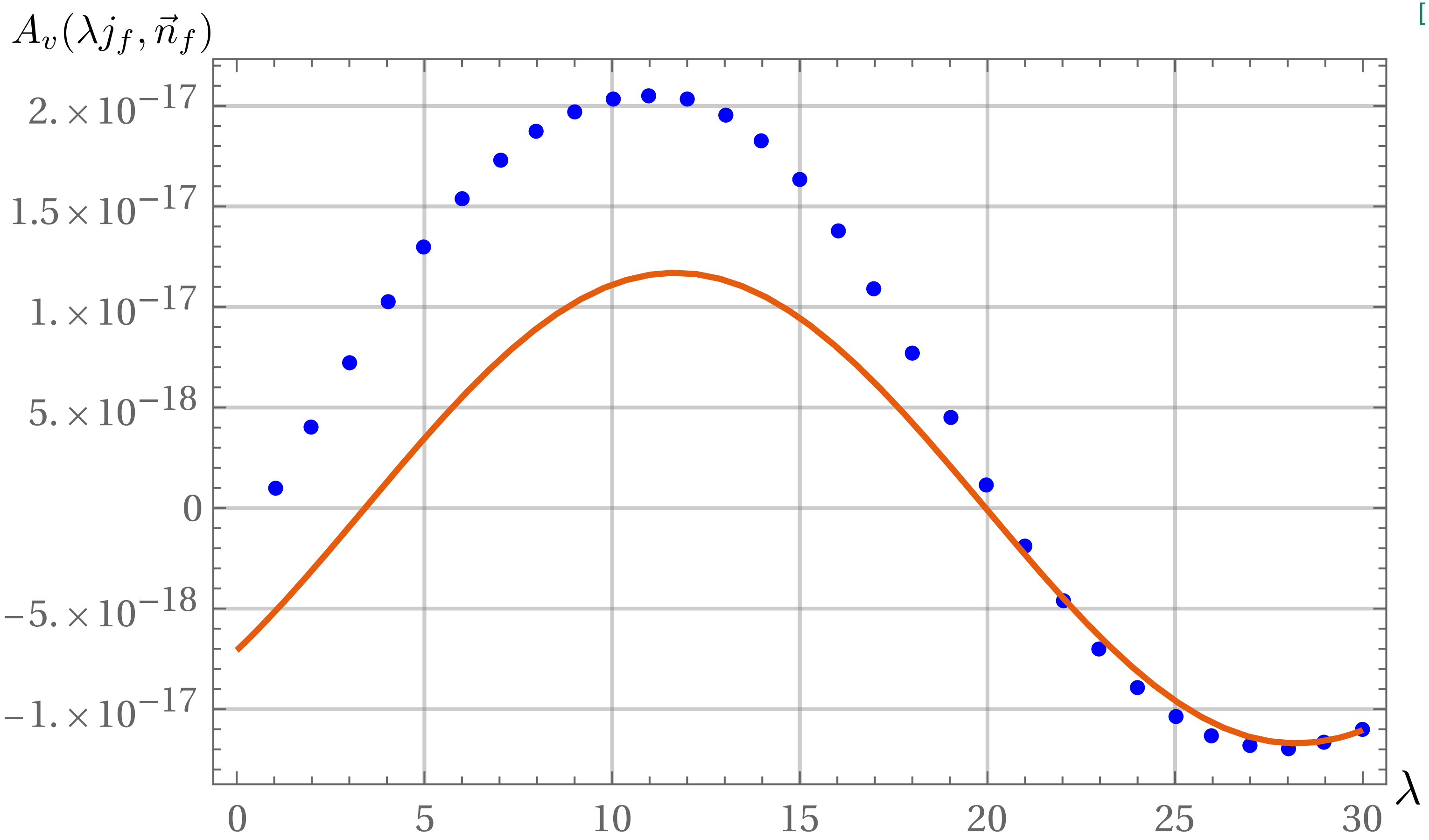
[P.D. M. Fanizza, G. Sarno, S. Speziale – 2019]

$$A_v(\lambda j_f, \vec{n}_f) = \frac{1}{\lambda^{12}}(N_1 e^{i\lambda S_R} + N_2 e^{-i\lambda S_R}) + O(\lambda^{-13})$$

Applications – Single vertex asymptotic

$$A_v(j_f, \vec{n}_f) = \sum_{i_e} A_v(j_f, i_e) \prod_e c_{i_e}(\vec{n}_f) = \frac{1}{\lambda^{12}}(N_1 e^{i\lambda S_R} + N_2 e^{-i\lambda S_R}) + O(\lambda^{-13})$$

[P.D. M. Fanizza, G. Sarno, S. Speziale – 2019]
[F. Gozzini – 2021]

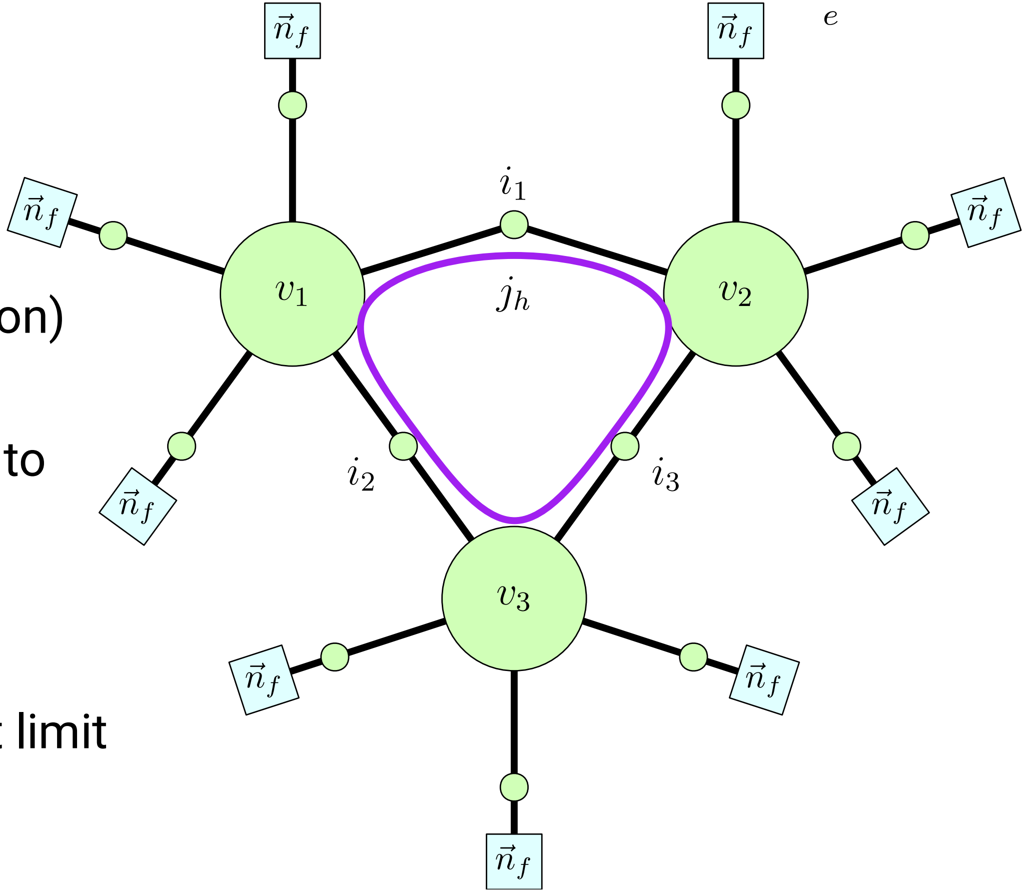


Applications – Many vertices asymptotic

$$A_{\Delta_3}(j_f,\vec{n}_f) = \sum_{i_e}\sum_{i_h}\sum_{j_h}(2j_h+1)A_{v_1}(j_f,i_e;j_h,i_{h_1},i_{h_2})A_{v_2}(j_f,i_e;j_h,i_{h_2},i_{h_3})A_{v_3}(j_f,i_e;j_h,i_{h_3},i_{h_1})\prod_e c_{i_e}(\vec{n}_f)$$

Flatness problem

- Uniform rescaling of the boundary spins
- Coherent boundary data (symmetric configuration)
- Flat and curved bulk geometry
- Exponential suppression of data corresponding to curved bulk geometries
- Problem!
  - Revise the model?
  - Indication that just large spins is not the right limit



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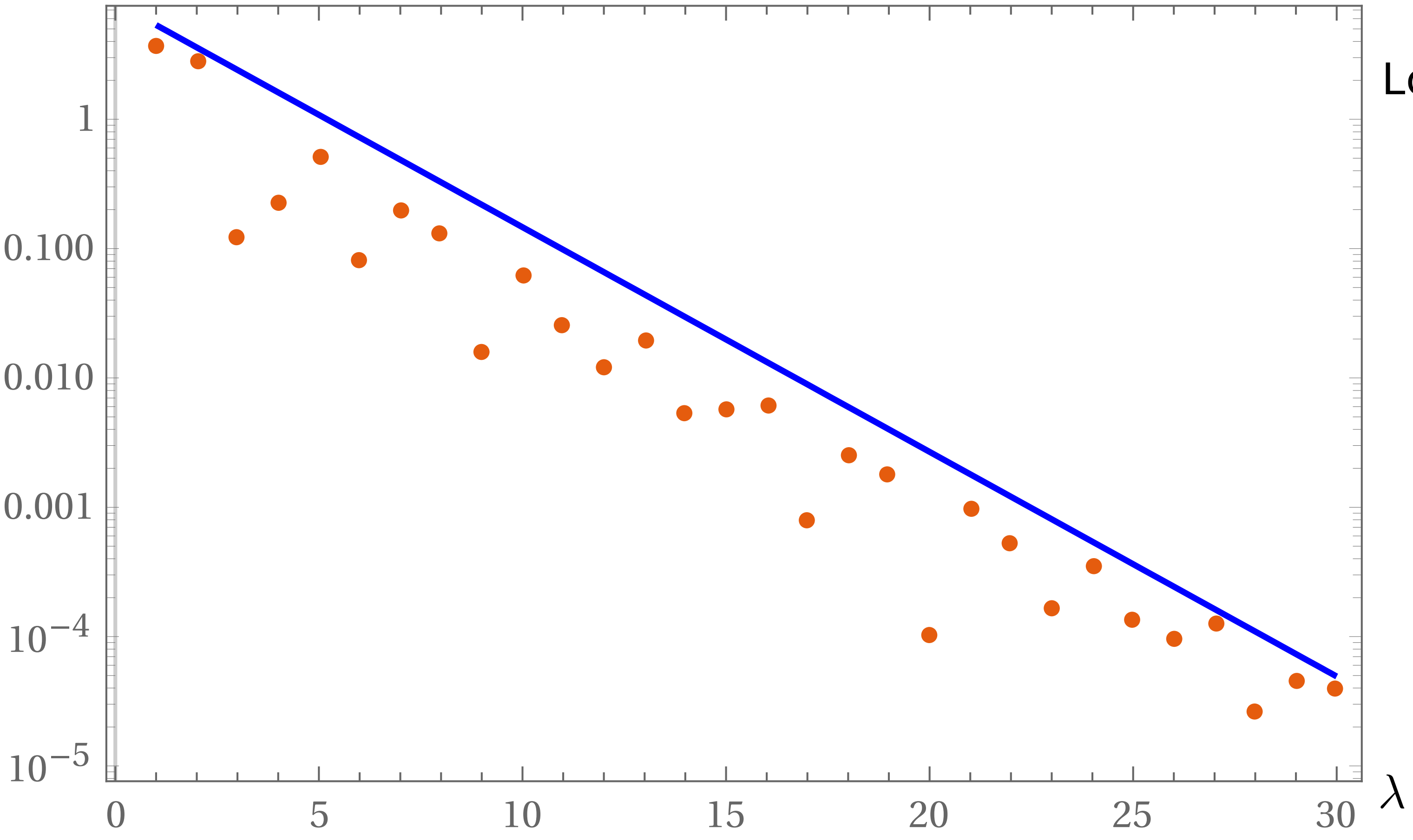


Log-Linear plot

[F. Gozzini – 2021]

[P.D. G. Sarno
F. Gozzini – 2021]

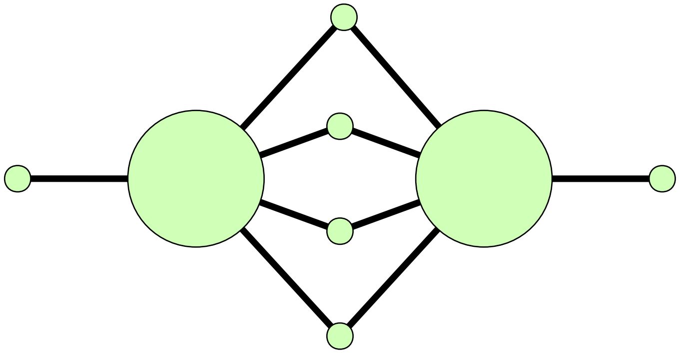
Applications – Radiative corrections

$$A_{rc} = \sum_{j_h}^{K} \left(\prod_{f \in h} (2j_f + 1) \right) \sum_{i_{h_e}} \left(\prod_{h_e} (2i_{h_e} + 1) \right) A_v^2(j_h, j_f, i_{h_e}, i_e)$$

Radiative corrections

- UV finite and IR divergent
- Important to study the continuum limit
- Cutoff K on the bulk spins
- Many complementary estimates ($\log K \leq A_{rc} \leq K^9$)
- First numerical estimates

[A. Riello 2013]
[P.D. 2018]



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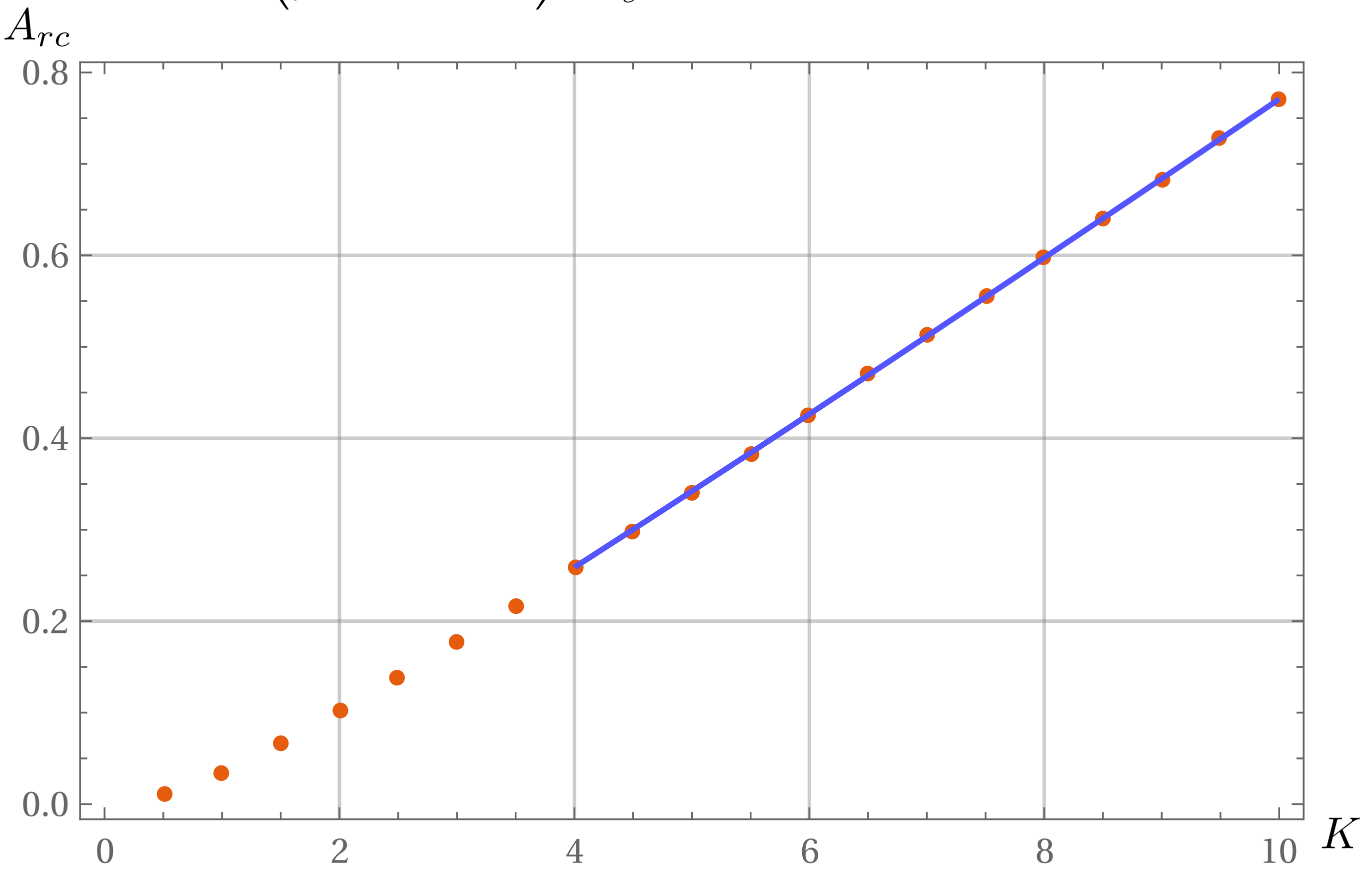

$\gamma = 0.1$

[P. Frisoni, F. Gozzini, F. Vidotto – 2022]

Conclusions

Spin foam (EPRL model) is a concrete implementation for a background independent Lorentzian dynamics for LQG

Technically very challenging. Invented new methods

Numerical framework is available and accessible to everyone

Applications: explore some open questions of the theory

Applications to cosmology are w.i.p. (a lot of vertices, Monte Carlo)