Pietro Donà

Center for Space, Time and the Quantum, Marseille
dona.pietro@gmail.com

Numerical evaluation of spin foam transition amplitudes

Loops 22 – ESN Lyon – 21st July 2022

We have a cool new toy, and I invite you to play with it!

Nuclear Physics B
Volume 799, Issues 1–2, 11 August 2008, Pages 136-149

LQG vertex with finite Immirzi parameter

Jonathan Engle [a], Etera Livine [b], Roberto Pereira [a], Carlo Rovelli [a]

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Abstract

We extend the definition of the “flipped” loop-quantum-gravity vertex to the case of a finite Immirzi parameter γ. We cover both the Euclidean and Lorentzian cases. We show that the resulting dynamics is defined on a Hilbert space isomorphic to the one of loop quantum gravity, and that the area operator has the same discrete spectrum as in loop quantum gravity. This includes the correct dependence on γ, and, remarkably, holds in the Lorentzian case as well. The *ad hoc* flip of the symplectic structure that was required to derive the flipped vertex is not anymore required for finite γ. These results establish a bridge between canonical loop quantum gravity and the spinfoam formalism in four dimensions.

Previous article in issue   Next article in issue

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[CREDITS to the majority of the people in this room and many other...]

Lorentzian EPRL model

Canonical LQG dynamics, path integral, sum over histories, background independent, Lorentzian
EPRL is the state-of-the-art

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connection with (discrete) GR,
graviton propagator, bubbles
divergences, cosmology, black holes
tunneling

Main open questions

Recover Einstein equations? (my opinion)

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Tools for computations

Saddle point techniques
Numerical methods

Simone Speziale (*motivator*)

Giorgio Sarno (*pioneer*)

Francesco Gozzini (*master coder*)

Pietropaolo Frisoni (*maintainer*)

[S. Speziale - 2017]

[P.D. M. Fanizza, G. Sarno S. Speziale – 2018]

sl2cfoam [P.D. G. Sarno – 2018]

[P.D. M. Fanizza, G. Sarno S. Speziale – 2019]

[P.D. G. Sarno F. Gozzini – 2020]

[P.D. G. Sarno F. Gozzini – 2021]

sl2cfoam-next [F. Gozzini – 2021]

[P. Frisoni, F. Gozzini, F. Vidotto – 2022]

[P.D., P. Frisoni – 2022]

[P.D., E. Wilson-Ewing, P. Frisoni – 2022]

[P. Frisoni, F. Gozzini, F. Vidotto – 2022]

MADE IN ITALY

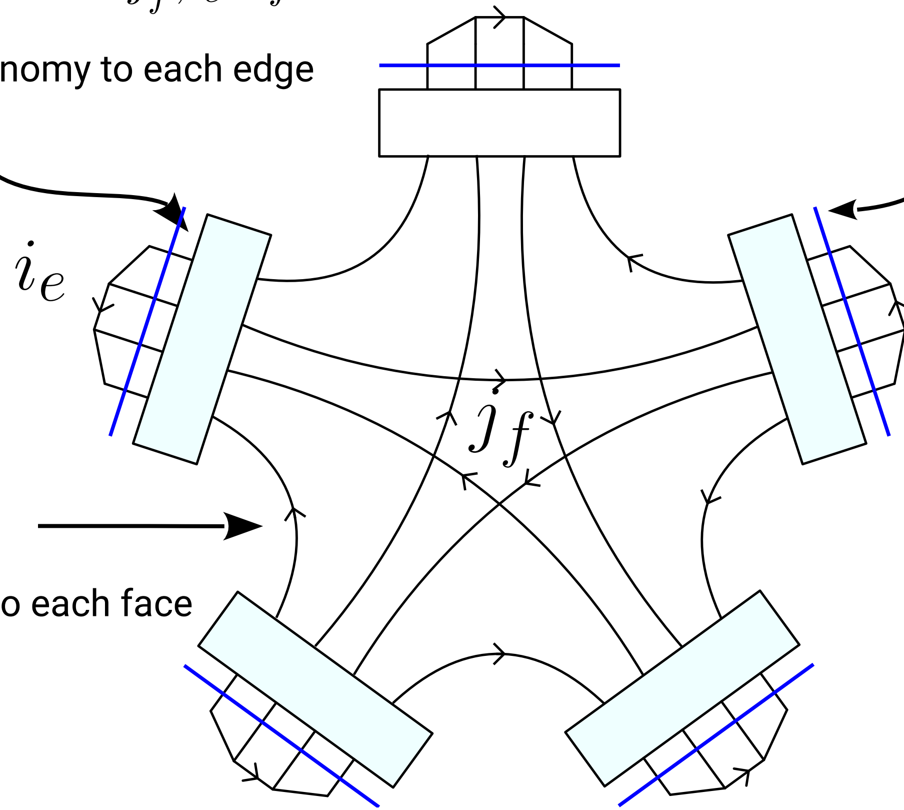
(Simplicial) EPRL spin foam model

$$A_\Delta = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v\left(j_f,\ i_e\right)$$

Associate a *SL(2,C)* holonomy to each edge

$Y_\gamma : j_f \to (\gamma j_f, j_f)$

+ project on the lowest spin sector.
Implements **simplicity constraints**



$D^{\gamma j_f j_f}_{j_f m j_f n}(g_t^{-1} g_s)$

Associate a unitary irrep to each face

(Simplicial) EPRL spin foam model

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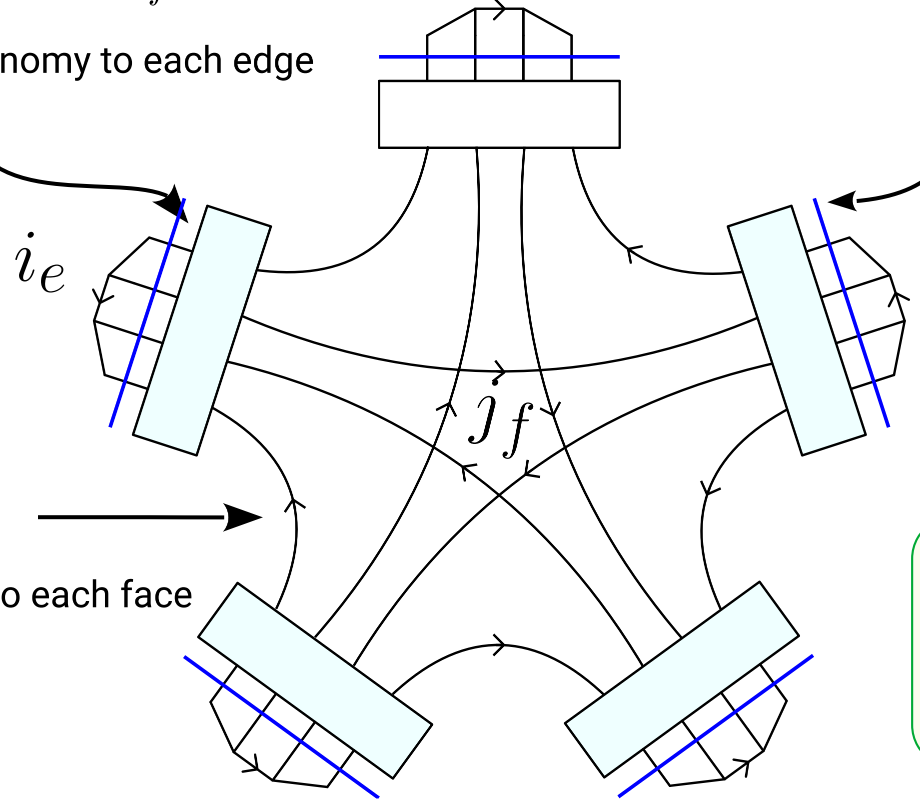
Regularize and **integrate** over all possible holonomies [J. Engle 2008]



$$D^{\gamma j_f j_f}_{j_f m j_f n}(g_t^{-1} g_s)$$

Associate a unitary irrep to each face

Put the amplitude in a computer

(Simplicial) EPRL spin foam model

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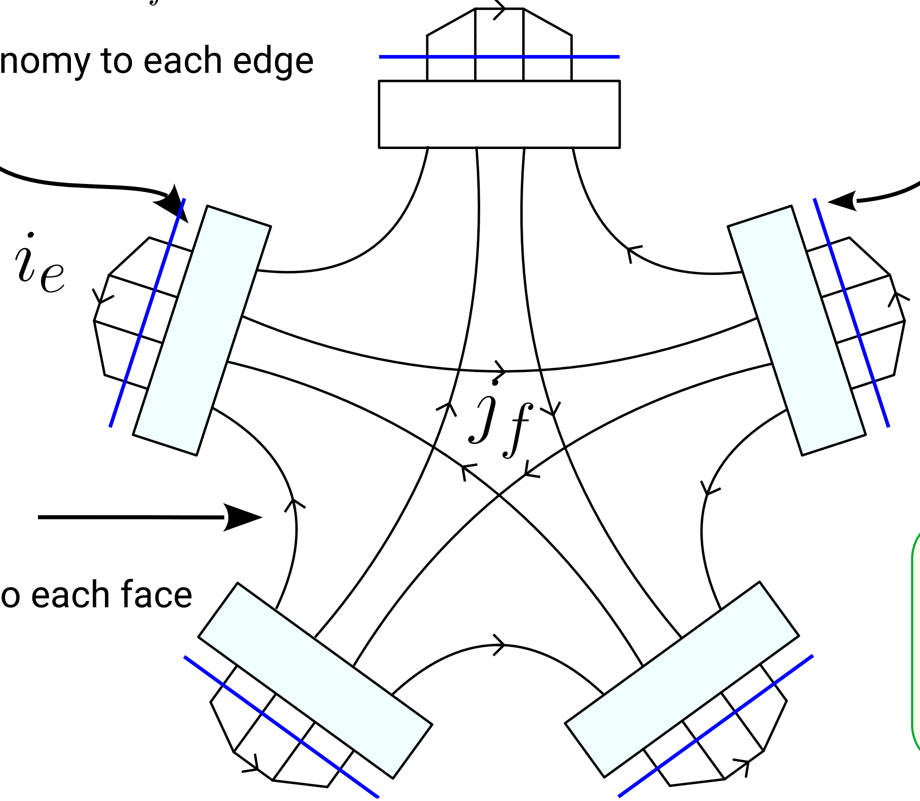


Regularize and **integrate** over all possible holonomies [J. Engle 2008]

4 copies of non compact **6** dimensional group of highly oscillating functions

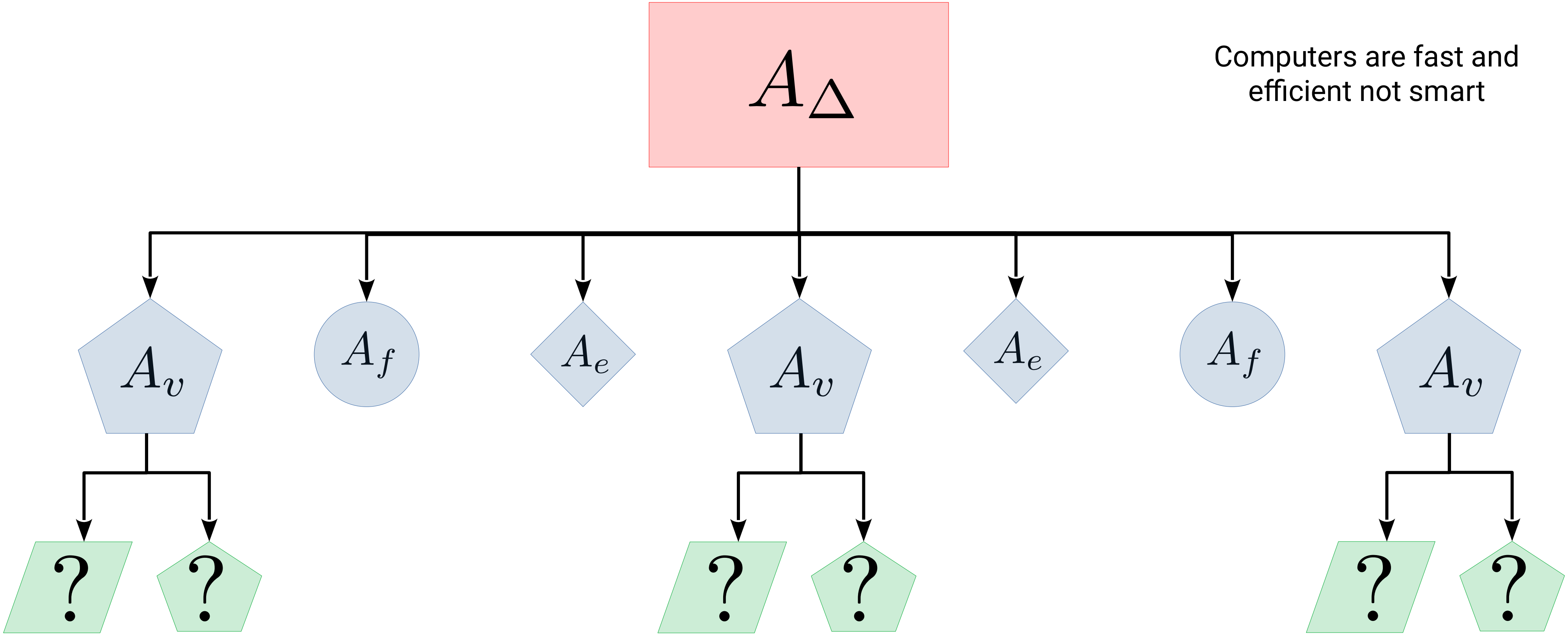
$$D^{\gamma j_f j_f}_{j_f m j_f n}(g_t^{-1} g_s)$$

Associate a unitary irrep to each face

Brute force integration is doomed to fail! New strategy?

Divide and conquer

Computers are fast and efficient not smart



Divide and conquer: How?

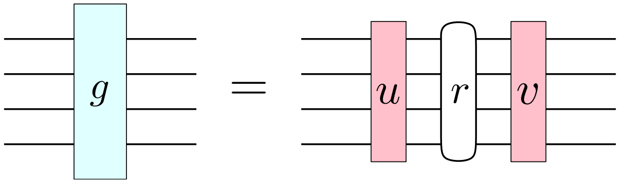
[S. Speziale - 2017]

Booster decomposition of A_v

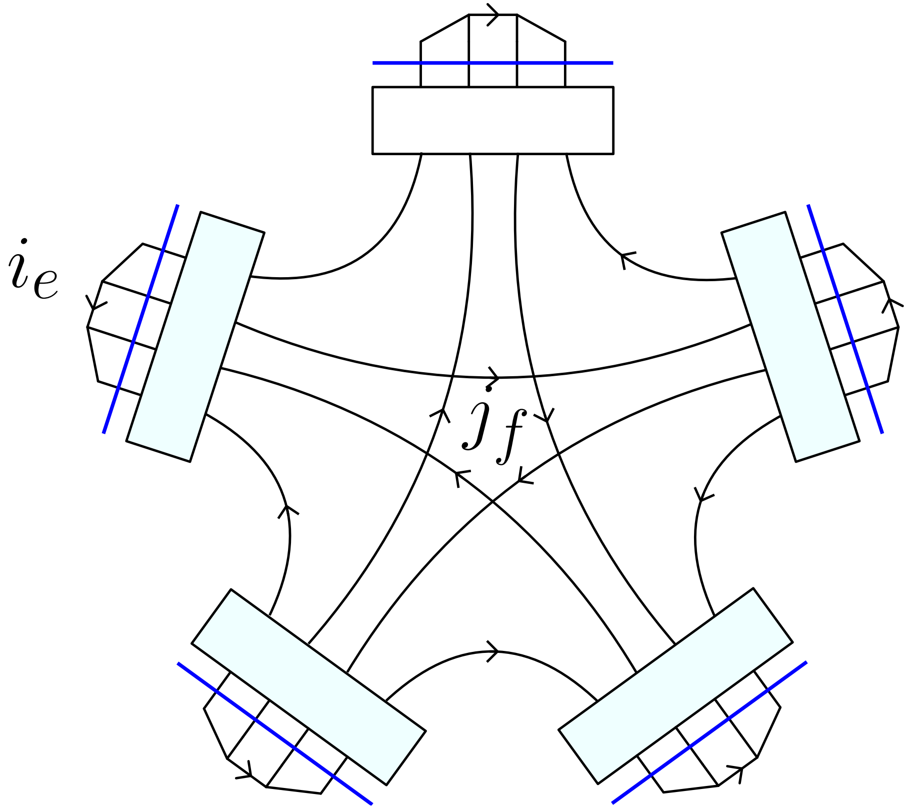
Cartan parametrization

$$g = u e^{-i\frac{r}{2}\sigma_3} v^\dagger$$

$$u, v \in SU(2) \quad r \in [0, +\infty)$$



explicit integration
over SU(2)



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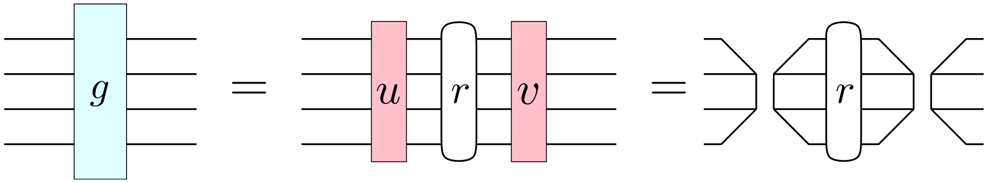
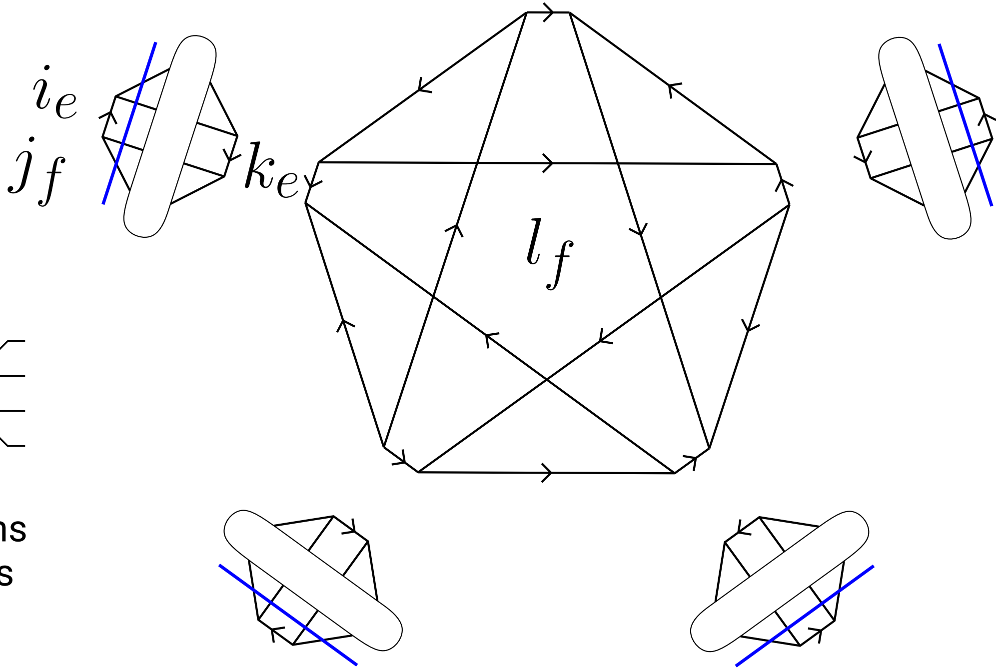


explicit integration over SU(2)

Booster functions and intertwiners

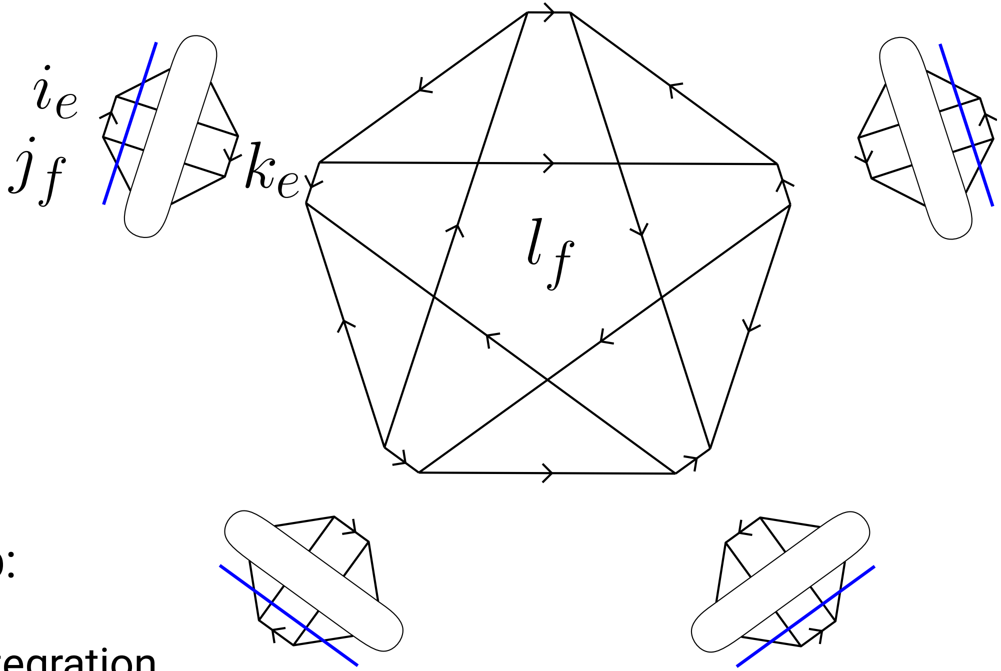


Divide and conquer: How?

[S. Speziale - 2017]

$$A_v(j_f, i_e) = \sum_{l_f = j_f}^{\infty} \sum_{k_e} \left(\prod_e (2k_e + 1) B_4(j_f, l_f, i_e, k_e) \right) \{15j\}(l_f, k_e)$$

A_v is a linear combination of $SU(2)$ invariants weighted by booster functions



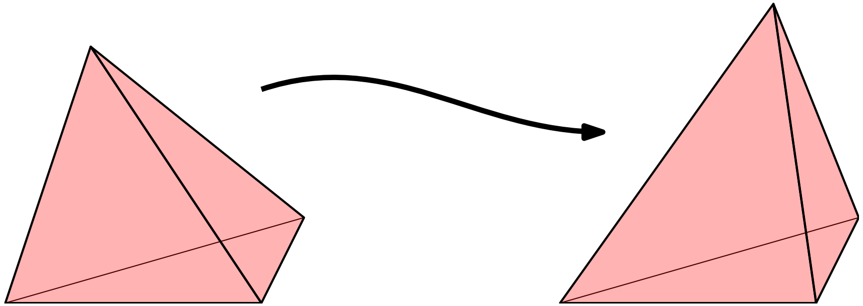
Remnants of non compactness of the group:

- Sum over virtual spins
- Four factorized one dimensional unbounded integration

Booster function

$$B_4(l_f, j_f, i_e, k_e) \equiv \sum_{m_f} \begin{pmatrix} l_f \\ m_f \end{pmatrix}^{(i_e)} \left(\int_0^\infty d\mu(r) \prod_f d^{(\gamma j_f, j_f)}_{l_f j_f m_f}(r) \right) \begin{pmatrix} j_f \\ m_f \end{pmatrix}^{(k_e)}$$

Interesting geometrical interpretation



Self-dual part of a gamma-simple bivector
(electric field + non trivial magnetic field)
Non-minimal $SU(2)$ sector → non-canonical frame

[P.D., P. Martin-Dussaud, M. Fanizza, S. Speziale – 2020
+ Simone Speziale's talk on Monday]

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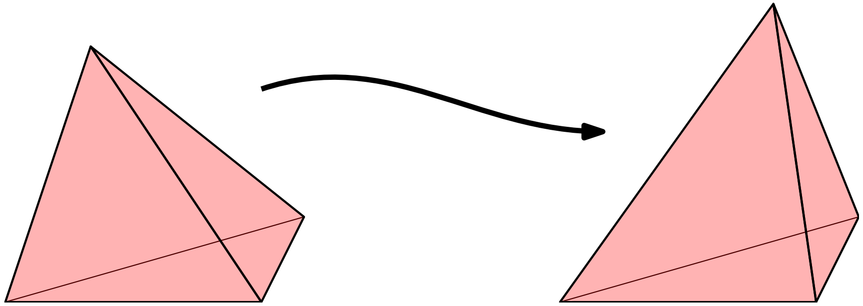

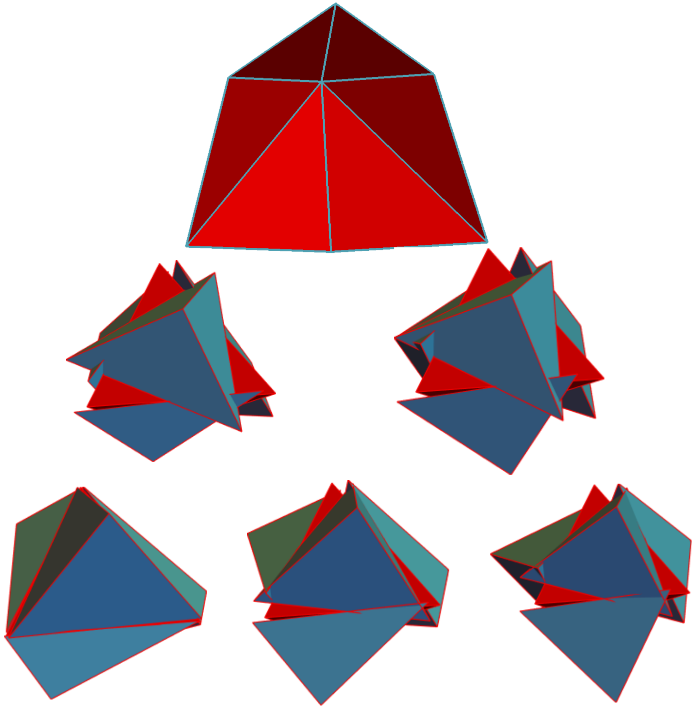

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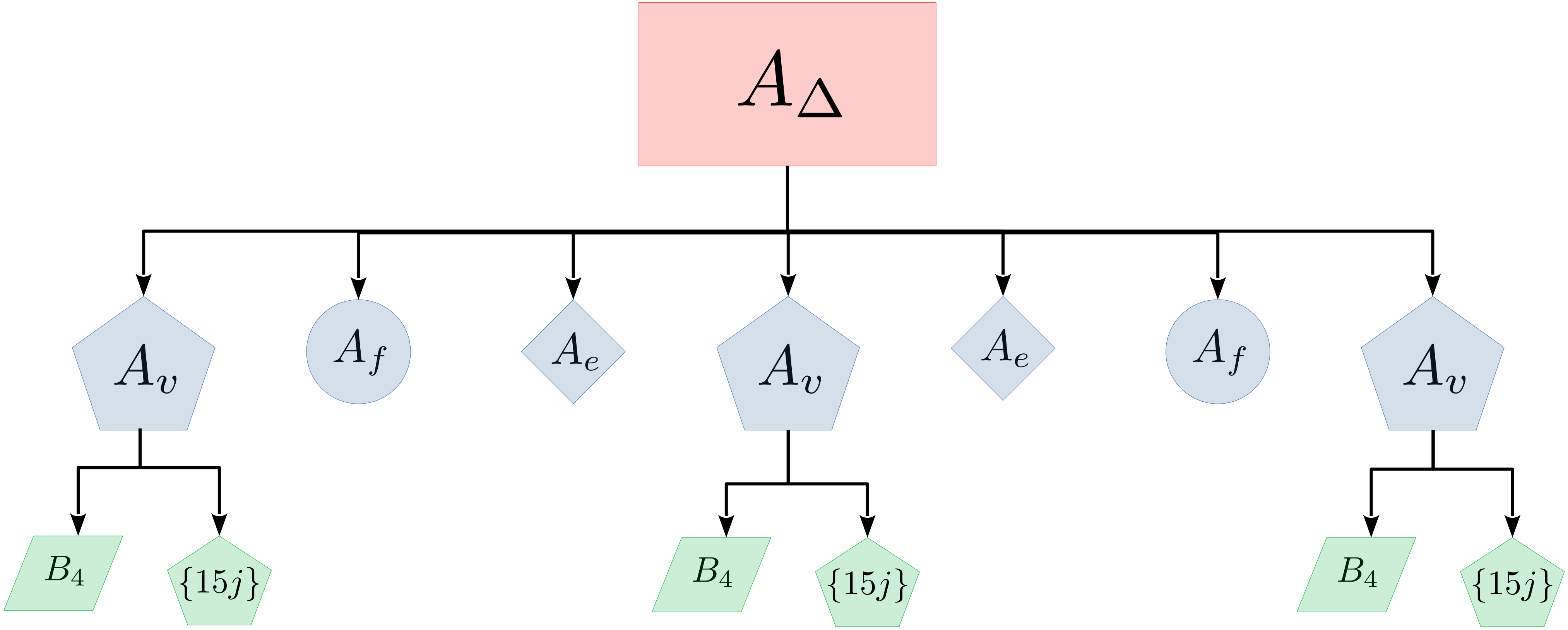
Divide and conquer



Series of
repeatable tasks!

Just code it!

$$A_v(j_f, i_e) = \sum_{l_f = j_f}^{\infty} \sum_{k_e} \left(\prod_e (2k_e + 1) B_4(j_f, l_f, i_e, k_e) \right) \{15j\}(l_f, k_e)$$

sl2cfoam & sl2cfoam-next

[P. D., G. Sarno – 2018]
[F. Gozzini - 2021]

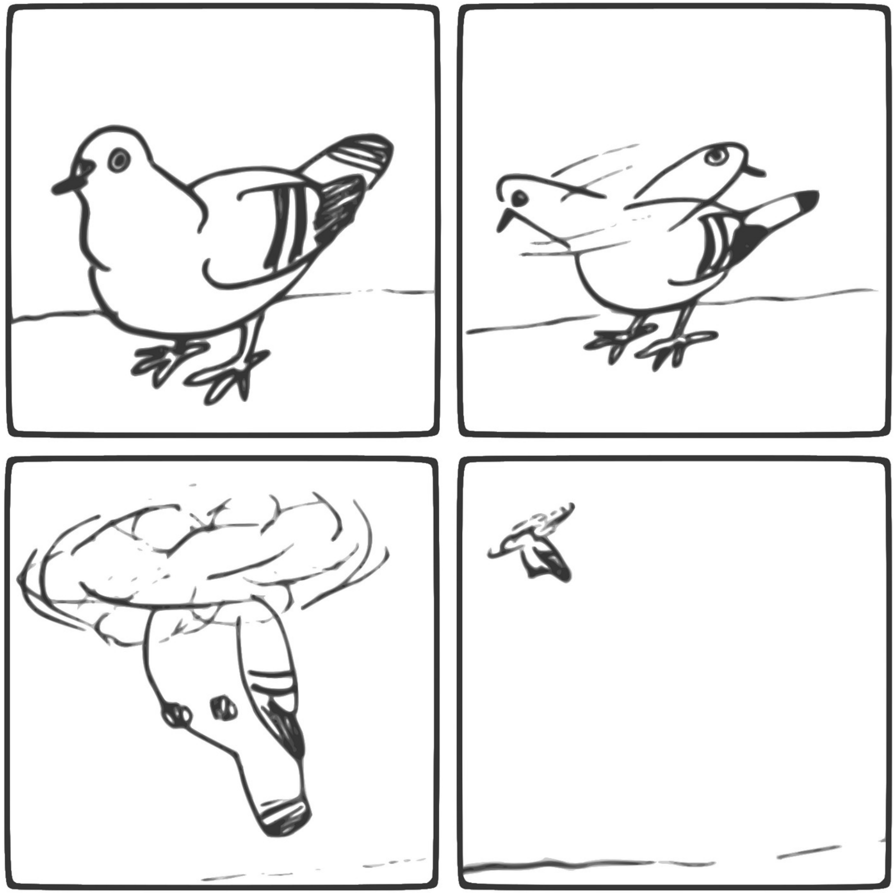
Presented at LOOPS19

Proof of concept (is it possible?)

Many technical limitation

The structure was the correct one!

When your program is a complete mess, but it does its job



sl2cfoam & sl2cfoam-next

[P. D., G. Sarno – 2018]
[F. Gozzini - 2021]

Advantages

- Open source – bit.ly/sl2cfoam-next
- Fast – C code
- Modular & Scalable – mix & match
- Optimized for HPC – parallelizable, GPU
- User friendly – Julia interactive interface

sl2cfoam & sl2cfoam-next

[P. D., G. Sarno – 2018]
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- Modular & Scalable – mix & match
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Disadvantages

- Resource demanding – costly
- Unavoidable approximation – error control

Numerical recipe with just three ingredients

$$A_v(j_f, i_e) = \sum_{l_f = j_f}^{\infty} \sum_{k_e} \left(\prod_e (2k_e + 1) B_4(j_f, l_f, i_e, k_e) \right) \{15j\}(l_f, k_e)$$

SU(2) invariants

- Well studied (spectroscopy, nuclear physics, solid state)
- We implemented {15j} of the first kind (sum of {6j} symbols)
- WIGXJPF and FASTWIGXJ [H. T. Johansson and C. Forssén - 2015]

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Booster functions

One dimensional integral (mapped in [0,1])

Highly oscillating functions (arbitrary precision)

Adaptive Gauss-Kronrod quadrature method

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[H. T. Johansson and C. Forssén - 2015]

Truncation parameter (unbounded sums)

Necessary approximation (uniform truncation)

Good approximation (convergent sums)

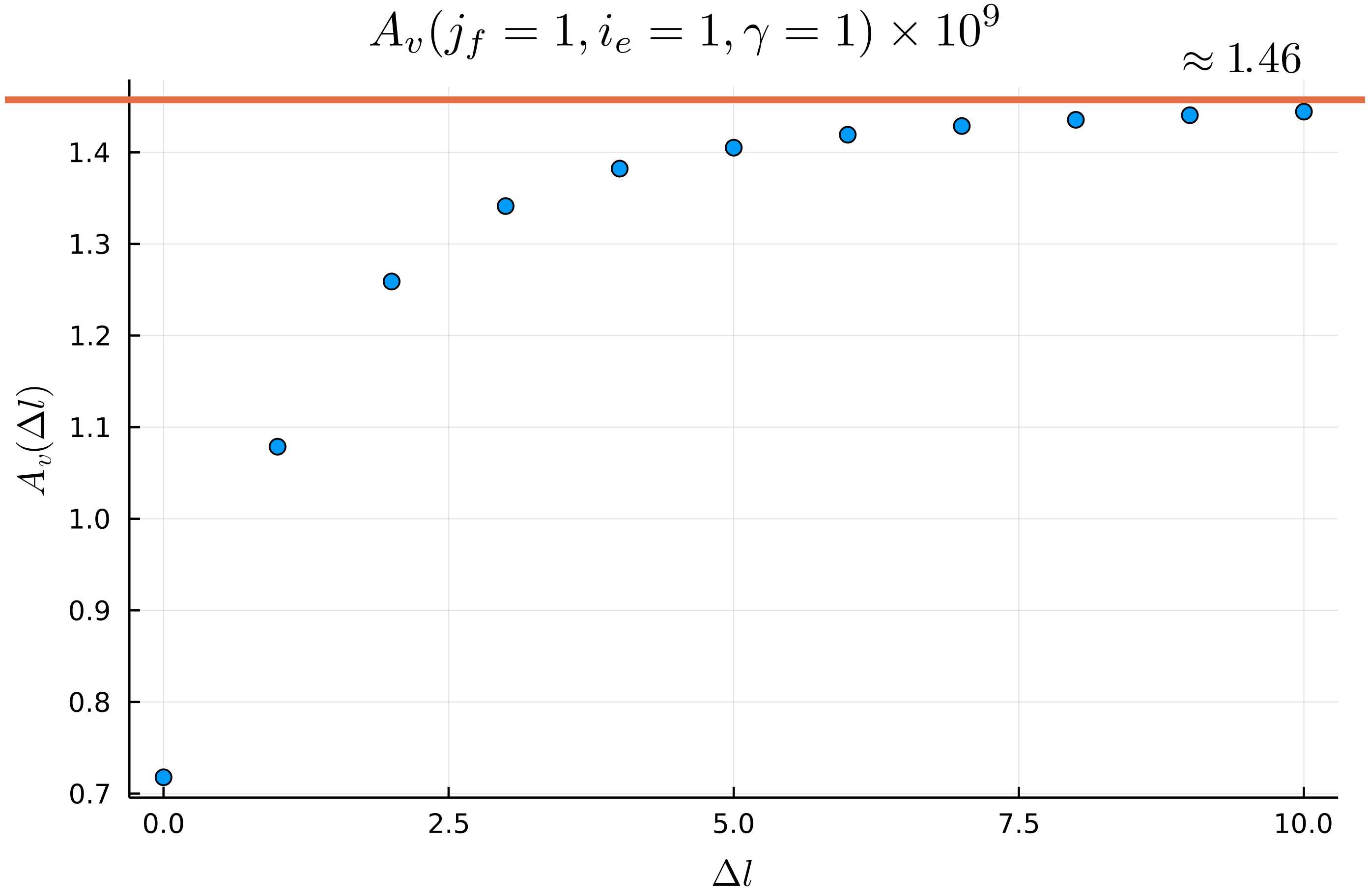
Weak approximation (no error estimate)

Beyond the approximation
(convergence acceleration techniques)

$$\sum_{l=j_f}^{\infty} \rightarrow \sum_{l=j_f}^{j_f+\Delta l}$$

Example: convergence of the truncation

[P.D., P. Frisoni – 2022]



Few lines of code

Tutorial
[P.D., P. Frisoni – 2022]

```
using SL2Cfoam
Immirzi = 1.2
data_folder = "path_data_folder"
configuration = SL2Cfoam.Config(VerbosityOff, VeryHighAccuracy, 100, 0)
SL2Cfoam.cinit(data_folder, Immirzi, configuration)
boundary_spins = ones(10)
Dl = 10
Av = vertex_compute(boundary_spins, Dl)
```

Initialization and calculation

Production stage – three applications

1. Single vertex in the large spin limit

2. Three vertices in the large spin limit

3. Bubbles and divergence estimates

Application - Single vertex in the large spin regime

Setting

One Lorentzian EPRL vertex with coherent boundary data

In the large spin regime (uniform rescaling) $j_f \to \lambda j_f$

Theorem:

- General data = exponentially decaying
- Vector Geometry = power law decaying with no oscillation
- Euclidean 4-simplex = power law and oscillation given by Regge Action
- Lorentzian 4-simplex = power law and oscillation given by Regge action with Immirzi parameter.

$$A_v \approx \frac{N_1}{\lambda^{12}} e^{i\lambda \sum_f \gamma j_f \theta_f + j_f \xi_f} + \frac{N_2}{\lambda^{12}} e^{i\lambda \sum_f -\gamma j_f \theta_f + j_f \xi_f}$$

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sl2cfoam

[P.D. M. Fanizza, G. Sarno, S. Speziale – 2019]

Very good qualitative and quantitative agreement

sl2cfoam

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Correct order of magnitude

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sl2cfoam-next

[F. Gozzini – 2021]

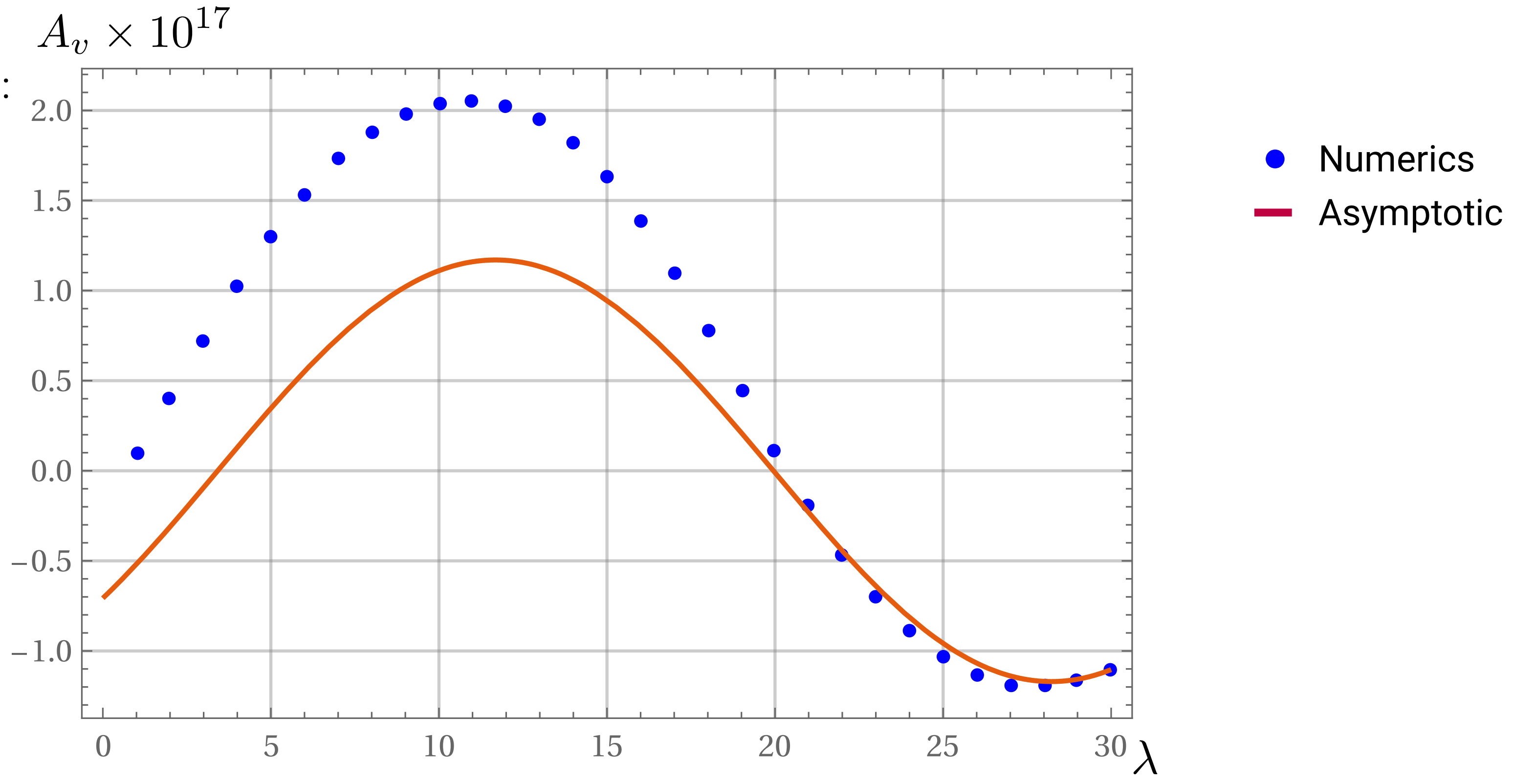
Very good qualitative and quantitative agreement

Application - Single vertex in the large spin regime

[F. Gozzini – 2021]

Symmetric boundary data:
Lorentzian 4-simplex,
integer areas, space-like
boundary – complicated

$$\gamma = 2 \qquad \Delta l = 8$$



Application - Three vertices in the large spin regime

Setting

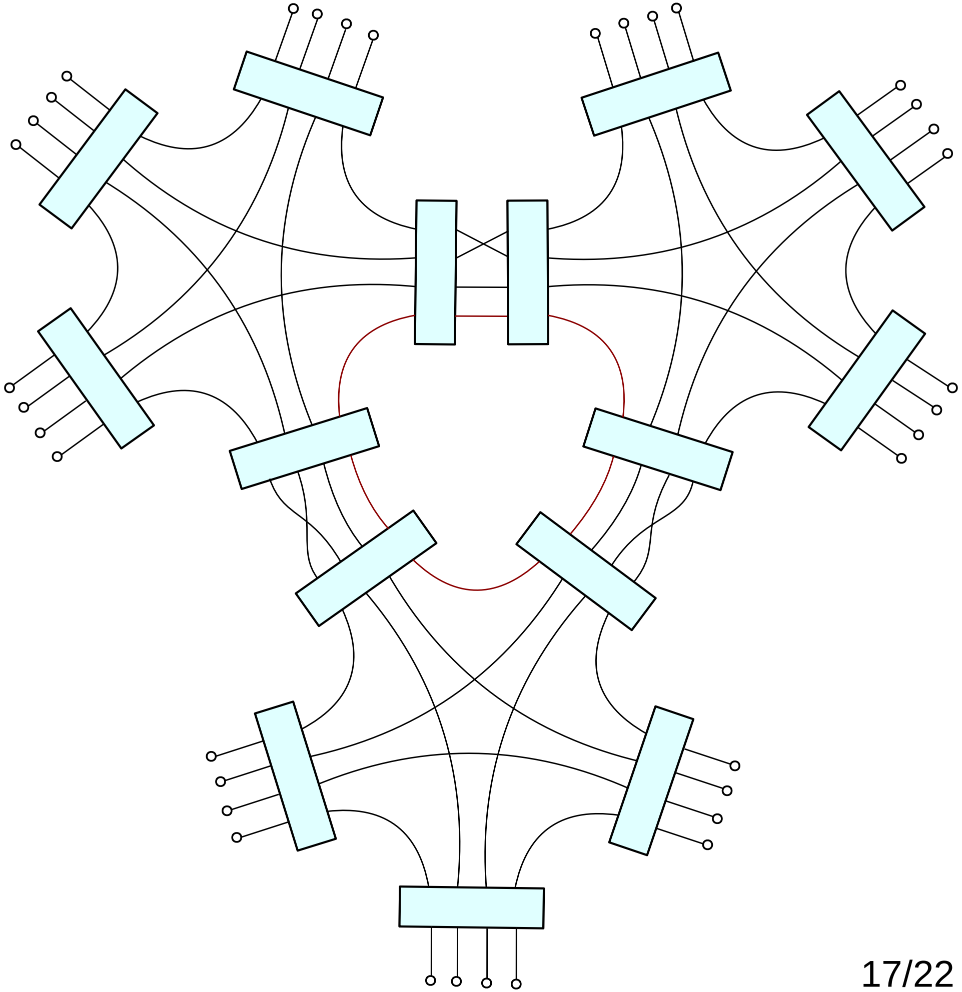
Three vertices, three bulk edges, one bulk face

In the large spin regime (uniform rescaling) $j_f \to \lambda j_f$

(non) flatness ~~problem~~:

- “Amplitude is exponentially suppressed iff boundary data is inconsistent with classical e.o.m.” [J. Engle yesterday]
- Exponential suppression of curved geometries with **special boundary** data
- Reignited discussion: not the correct semiclassical limit?

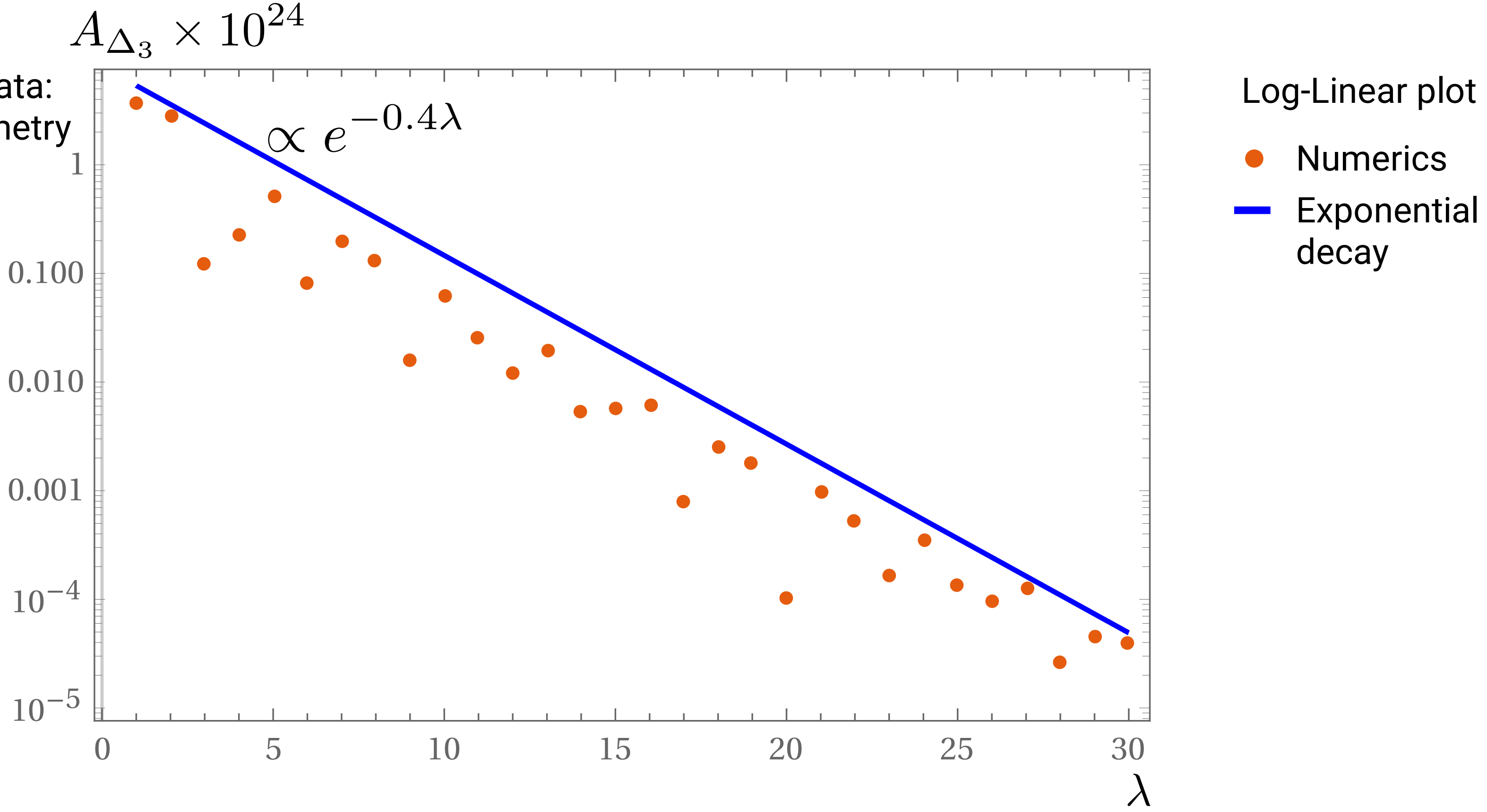
[S. Asante, B. Dittrich, H. Haggard, M. Han, H. Liu, C. Rovelli, J. Engle – Plenary talk by Engle yesterday]



Application - Three vertices in the large spin regime

Symmetric boundary data: curved (Euclidean) geometry

$\gamma = 2 \quad \Delta l = 2$



[P.D. G. Sarno, F. Gozzini – 2021]
[F. Gozzini – 2021]

Application - Bubbles and IR divergences

UV finite with potential IR divergences

[P. Frisoni's talk Friday afternoon for more details]

Large volume divergences from unbounded spin summations

$$A_\Delta = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v\left(j_f,\ i_e\right)$$

Continuum limit: GFT Renormalization? Control of diverging amplitudes

Analytical calculations of the degree of divergence of specific EPRL amplitudes are hard [A. Riello - 2013, P.D. - 2018]

No closed formula for the degree of divergence (examples are useful)

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Numerical estimate of radiative correction to EPRL spin foam edge

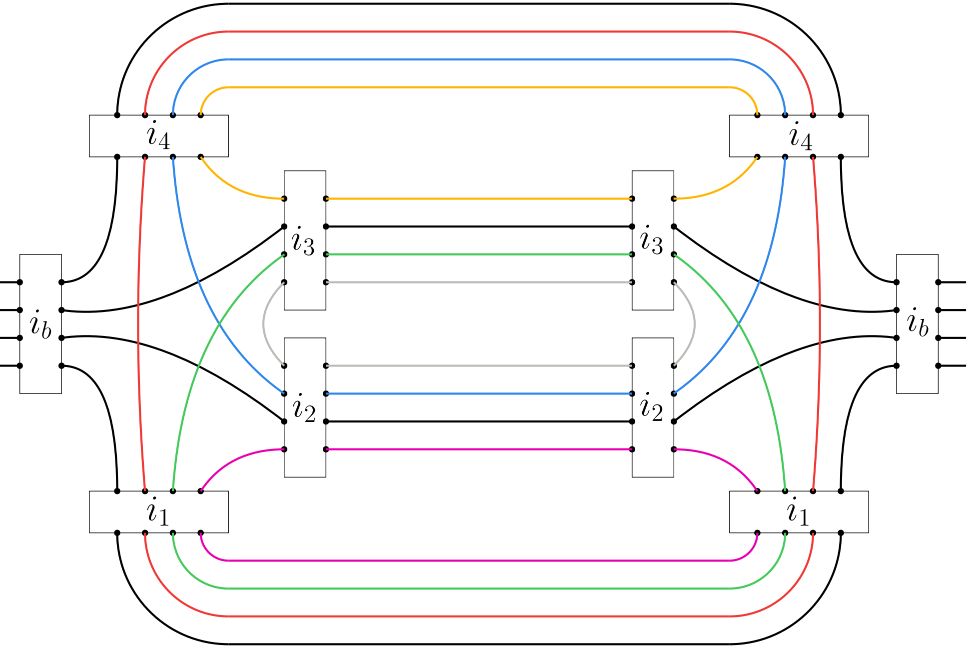
Homogeneous cutoff on all the bulk spins summations (K)

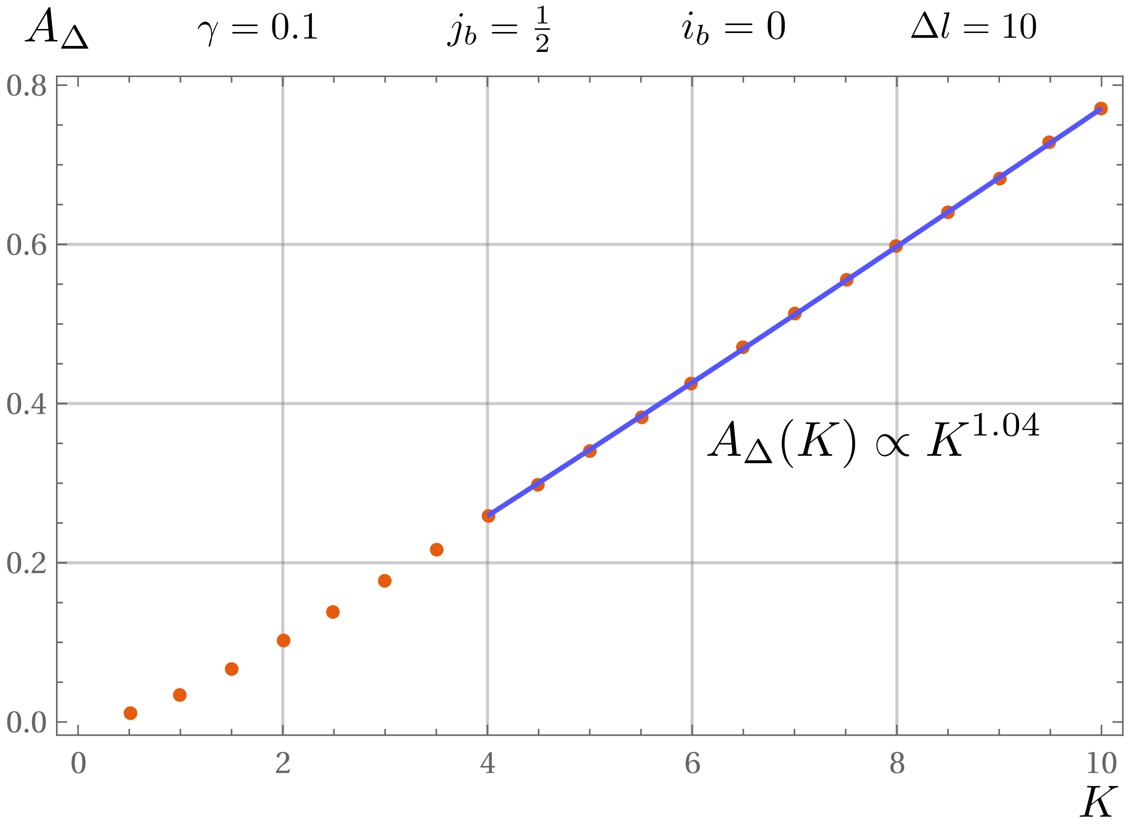
$$\sum_{j_f=0}^{\infty} \rightarrow \sum_{j_f=0}^{K}$$

All diagrams with:

- 2 vertices, 2 boundary edges, 4 boundary faces connecting both edges, trivial propagators

Application - Bubbles and IR divergences

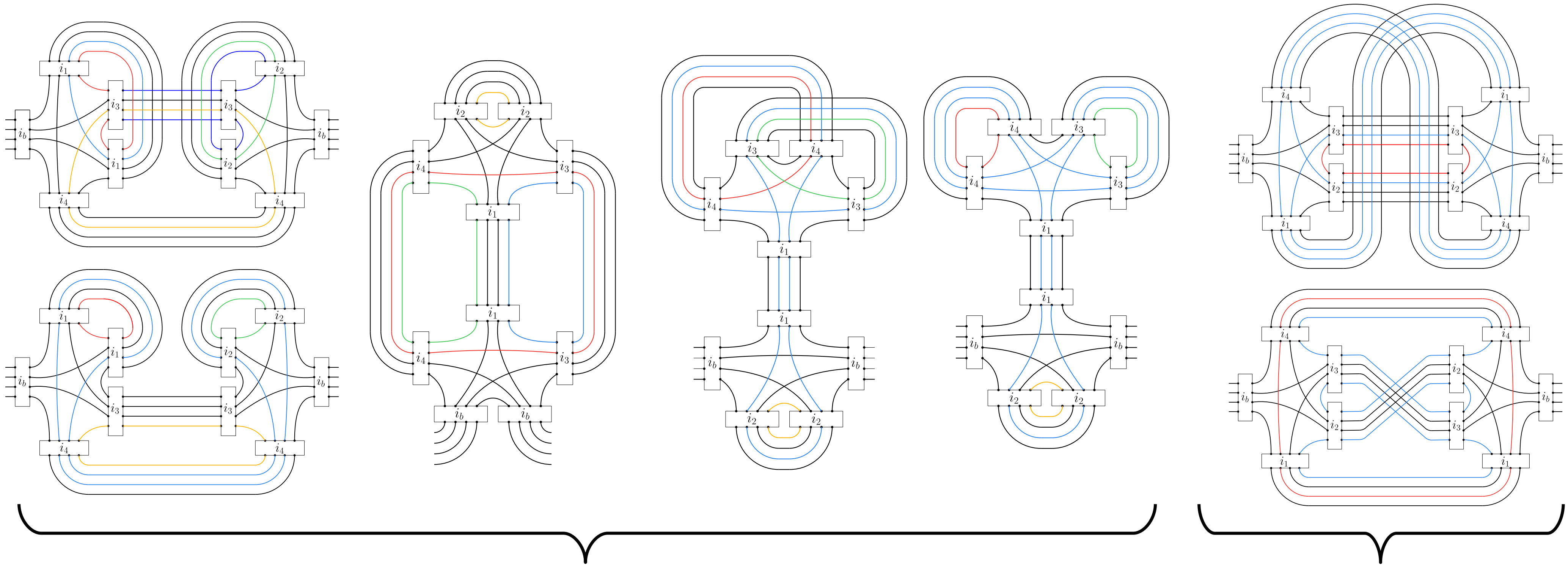




Numerical evidence for a linear divergence

[P. Frisoni, F. Gozzini, F. Vidotto – 2022]

Application - Bubbles and IR divergences



4 internal faces

2 internal faces

Numerical evidence for convergence of all these diagrams

[P.D., E. Wilson-Ewing, P. Frisoni – 2022]

(there is a second diagram with 6 unbounded faces but we do not have the resources to compute it now)

Conclusions

A numerical revolution for Covariant LQG

Many complementary frameworks are available (Effective spin foams, Lefschetz-thimbles, booster decomposition)

Different questions are solved more efficiently by different methods

S. Asante, B. Dittrich, H. Haggard, M. Han, Z. Huang, H. Liu, D. Qu, Y. Wan, P.D., G. Sarno, F. Gozzini, P. Frisoni

sl2cfoam-next

Open source, fast, scalable, modular framework to compute EPRL transition amplitudes

Numerous applications to answer open questions

Clear approximation, schemes to remove dependency on truncation

Future development

Improve documentation (installation guide)

Proposal for an hybrid scheme (similar to *chimera* in LQC) [S. Asante, J. Simão, S. Steinhaus]

Physically interesting calculations (Cosmological applications, black hole to white hole tunneling)

Efficient sum over bulk degrees of freedom (Monte Carlo techniques) [P. Frisoni, F. Gozzini, F. Vidotto] – Vidotto's talk on Tuesday + [P. Frisoni, C. Rovelli, F. Soltani] w.i.p.

We have a cool new toy, and I invite you to play with it!