Numerical study of spin foam theories

Pietro Dona

LOOPS'19

June 18th 2019

PennState
Eberly College
of Science

Thanks to my collaborators: E. Bianchi, M. Fanizza, F. Gozzini, G. Sarno, S. Speziale

Partially based on:
arXiv:1903.12624
Gen. Rel. Grav. 50, 127 (2018)
Class. Quant. Grav. 35, no. 17, 175019 (2018)
Class. Quant. Grav. 35, no. 4, 045011 (2018)

Spin foam theories

Covariant formulation of Loop Quantum Gravity dynamics

State of art is the EPRL-FK model [Engle, Pereira, Rovelli, Livine, Freidel and Krasnov - 2008]

Graviton Propagator
[Alesci, Bianchi, Han, Magliaro, Perini, Rovelli, Speziale, Zhang]

Connection with GR
[Barrett, Freidel, Han]

Extensions (Λ ,KKL, Timelike)
[Han, Lewandowski, Conrady]

Bubble Divergences
[Bonzom, P.D., Riello]

Linearized Gravity
[Han, Huang, Zipfel]

BH-WH transition
[Christodoulou, D'Ambrosio, Rovelli]

Spinfoam cosmology
[Bianchi, Gozzini, Vidotto]

Numerical evaluation
[P.D., Gozzini, Sarno]

Spin foam theories

Covariant formulation of Loop Quantum Gravity dynamics

State of art is the EPRL-FK model [Engle, Pereira, Rovelli, Livine, Freidel and Krasnov - 2008]

Graviton Propagator
[Alesci, Bianchi, Han, Magliaro, Perini, Rovelli, Speziale, Zhang]

Connection with GR
[Barrett, Friedel, Han]

Single vertex
[Han, P.D. Sarno, Speziale, Vidotto]

Continuum Limit
[Bahr, Dittrich, Han, Steinhaus]

Extensions (Λ ,KKL, Timelike)
[Han, Lewandowski, Conrady]

Bubble Divergences
[Bonzom, P.D., Riello]

Linearized Gravity
[Han, Huang, Zipfel]

Flatness Problem
[Freidel, Conrady, Bonzom, Hellmann, Kaminski]

BH-WH transition
[Christodoulou, D'Ambrosio, Rovelli]

Spinfoam cosmology
[Bianchi, Gozzini, Vidotto]

Numerical evaluation
[P.D., Gozzini, Sarno]

Cosine Problem
[Engle]

Spin foam theories

Covariant formulation of Loop Quantum Gravity dynamics

State of art is the EPRL-FK model [Engle, Pereira, Rovelli, Livine, Freidel and Krasnov - 2008]

Graviton Propagator
[Alesci, Bianchi, Han, Magliaro, Perini, Rovelli, Speziale, Zhang]

Connection with GR
[Barrett, Friedel, Han]

Single vertex
[Han, P.D. Sarno, Speziale, Vidotto]

Continuum Limit
[Bahr, Dittrich, Han, Steinhaus]

Extensions (Λ ,KKL, Timelike)
[Han, Lewandowski, Conrady]

Bubble Divergences
[Bonzom, P.D., Riello]

Linearized Gravity
[Han, Huang, Zipfel]

Flatness Problem
[Freidel, Conrady, Bonzom, Hellmann, Kaminski]

BH-WH transition
[Christodoulou, D'Ambrosio, Rovelli]

Spinfoam cosmology
[Bianchi, Gozzini, Vidotto]

Numerical evaluation
[P.D., Gozzini, Sarno]

Cosine Problem
[Engle]

Numerical methods will play a central role in the study of the spin foam amplitudes in the near future.

Plan of the talk

(0) Captatio Benevolentiae

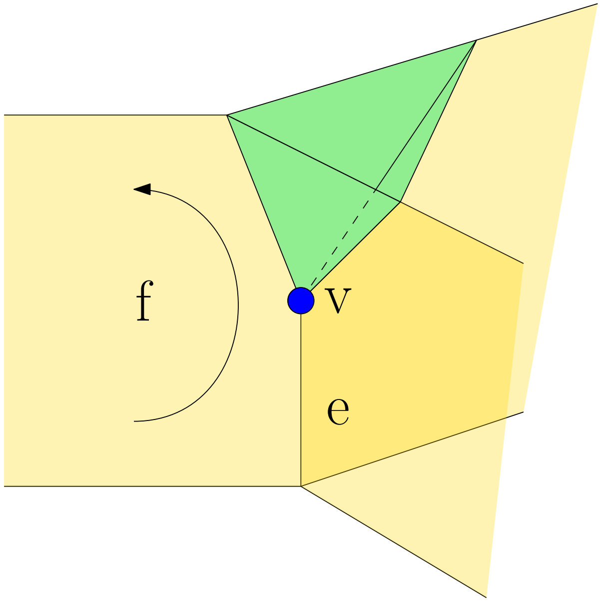
(1) The EPRL amplitude and the sl2cfoam library

(2) Vertex amplitude evaluation

(3) Transition amplitudes with more vertices

(4) Conclusion and Outlook

EPRL-FK model in one slide



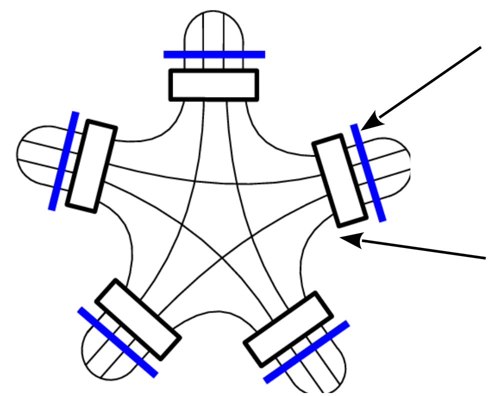
Transition amplitude between LQG states

$$W_{\mathcal{C}} = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f, i_e)$$

Trivial face and edge amplitude. Vertex amplitude:

[Bianchi, Regoli, Rovelli - 2010]

$$A_v\,(j_f, i_e) =$$

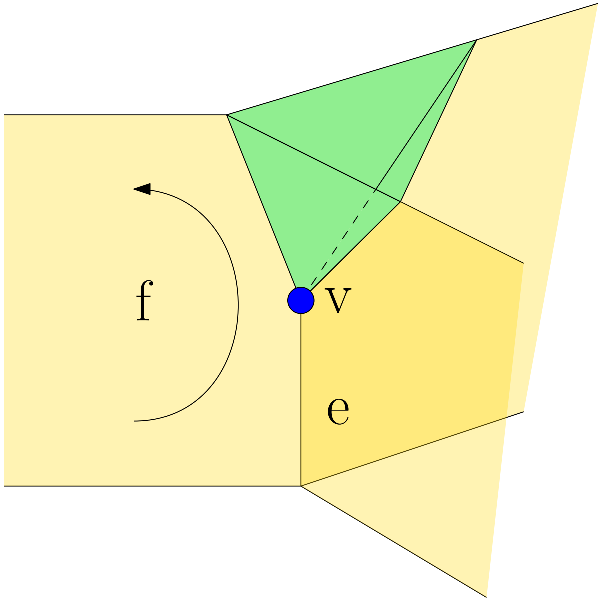


Y map :

$$j \longrightarrow (\rho, k) \stackrel{Y}{=} (\gamma j, j)$$

$$SL\,(2, \mathbb{C})$$

EPRL-FK model in one slide



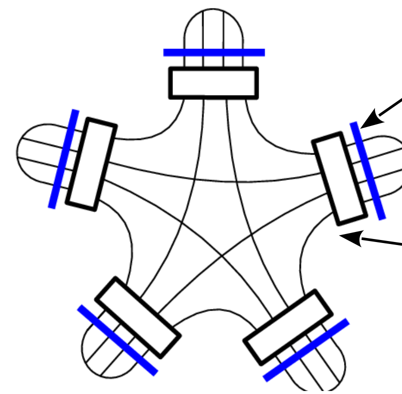
Transition amplitude between LQG states

$$W_{\mathcal{C}} = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f, i_e)$$

Trivial face and edge amplitude. Vertex amplitude:

[Bianchi, Regoli, Rovelli - 2010]

$A_v\,(j_f, i_e) =$



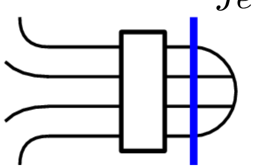
Y map :

$j \longrightarrow (\rho, k) \overset{Y}{=} (\gamma j, j)$

$SL\,(2, \mathbb{C})$

Decomposition:

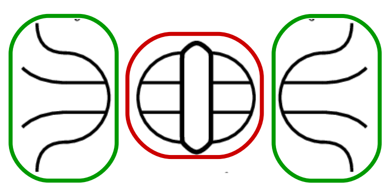
j_e, i



$u e^{\frac{r}{2}\sigma_3} v^{-1}$

=

l_e, k $\quad j_e, i$



$B_4\,(j_f, i_e, l_{fv}, k_{ev}; \gamma)$

Booster functions & SU(2) Intertwiners

[Speziale - 2016]

$$A_v\,(j_f,\, i_e) = \sum_{l_{fv}, k_{ev}} \left(\prod_{ev}{}' (2k_{ev}+1)\, B_4(j_{fv}, l_{fv}; i_{ev}, k_{ev}) \right) \{15j\}_v(l_{fv}, k_{ev})$$

The sl2cfoam library

[P.D. and Sarno – 2018]

Open source tools to compute the vertex amplitude (C library v2)

Maintained by G. Sarno and F. Gozzini

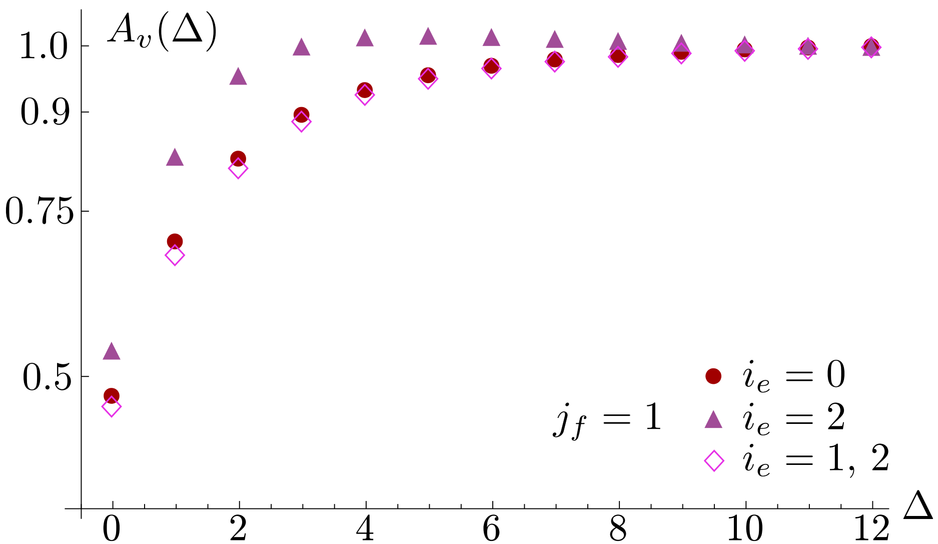
$$A_v\,(j_f,\,i_e) = \sum_{l_{fv},k_{ev}} \left({\prod_{ev}}' (2k_{ev}+1)\, B_4(j_{fv}, l_{fv}; i_{ev}, k_{ev}) \right) \{15j\}_v(l_{fv}, k_{ev})$$

Virtual spins sum

Unbounded but convergent sums

[Engle, Pereira – 2009]

Truncation $\quad l_f \leq j_f + \Delta$



EPRLs

SU(2) invariants:

Optimized algorithm to compute $\{6j\}$ symbols

[Johansson, Forssen – 2015]

Tested with large spin asymptotic

[D. , Fanizza, Sarno, Speziale – 2017]

Booster functions:

$SL\,(2,\mathbb{C})$ Clebsch–Gordan coefficients

Integration over the rapidity

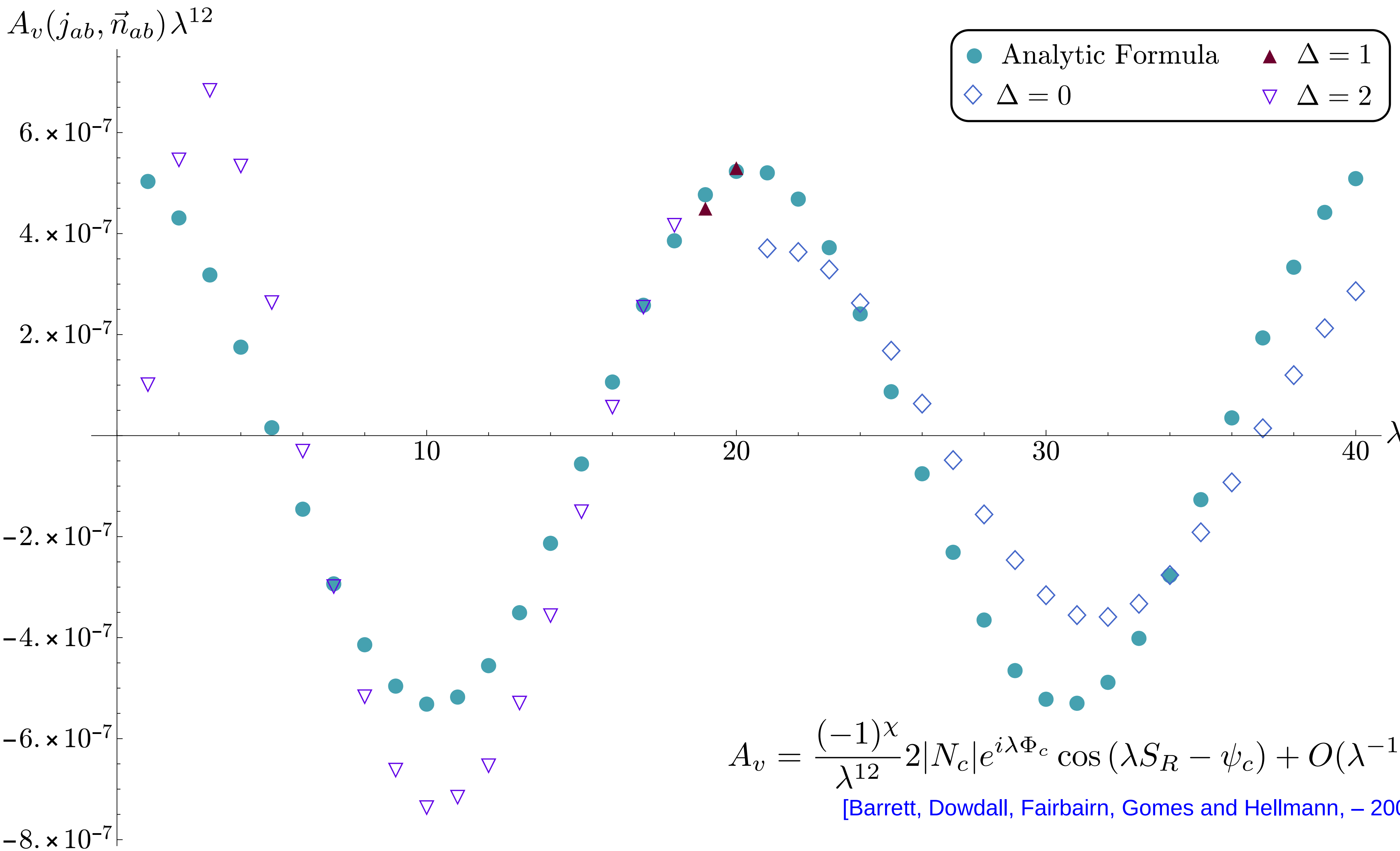
Arbitrary precision mathematics (interference)

Interesting geometric interpretation

[Fanizza, Martin-Dussaud, Speziale – to appear]

Single Vertex – Regular 4-simplex

[P.D. , Fanizza, Sarno, Speziale – 2019]

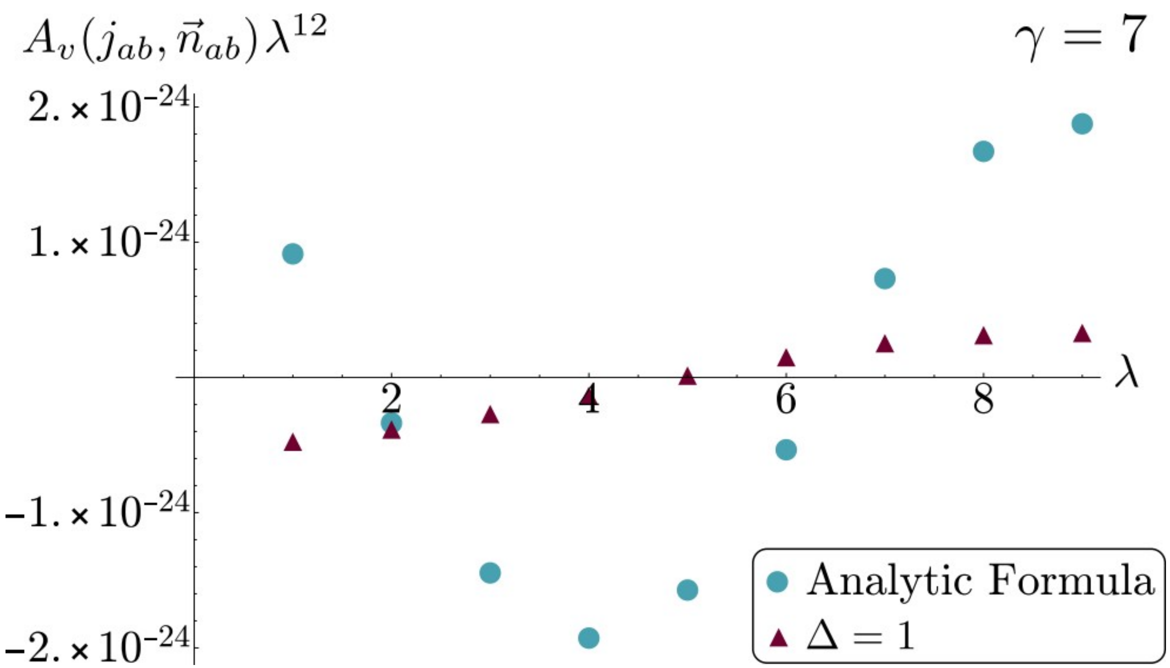


$$A_v = \frac{(-1)^\chi}{\lambda^{12}} 2|N_c| e^{i\lambda\Phi_c} \cos\left(\lambda S_R - \psi_c\right) + O(\lambda^{-13})$$

[Barrett, Dowdall, Fairbairn, Gomes and Hellmann, – 2009]

Single Vertex – Lorentzian 4simplex

[P.D. , Fanizza, Sarno, Speziale – 2019]



$$A_v = \frac{(-1)^{\chi+M}}{\lambda^{12}} 2|N_c| e^{i\lambda\Phi_c} \cos(\gamma\lambda S_R - \psi_c) + O(\lambda^{-13})$$

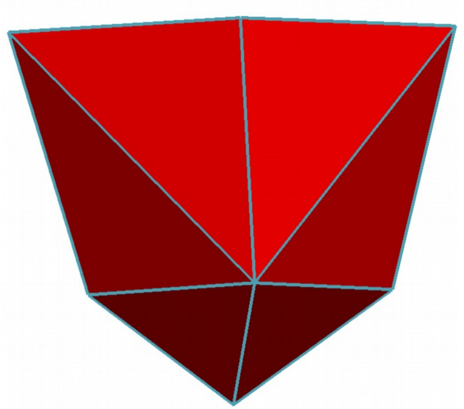
[Barrett, Dowdall, Fairbairn, Gomes and Hellmann, – 2009]

Right power law scaling

Evident dependence on γ

Problematic boundary data

Spins too large too soon



What did we learn?

Precise evaluation of the complete Lorentzian EPRL model are currently possible (address explicit physical questions)

The pure quantum regime $j_f \lessapprox 10$ is completely under control

Spin 1/2 truncation of the theory = 32 numbers (connection to RG)

[yesterday talk by Bahr, tensor network]

Technical upgrades

Recursion relations of SL(2,C) Clebsch–Gordan

Selection rules

Revisiting the asymptotic analysis

Different from the reconstruction theorem [P.D. , Fanizza, Sarno, Speziale – 2017/2019]

Immediate extension to KKL spin foam [Kaminski, Kisielowski, and Lewandowski - 2010]

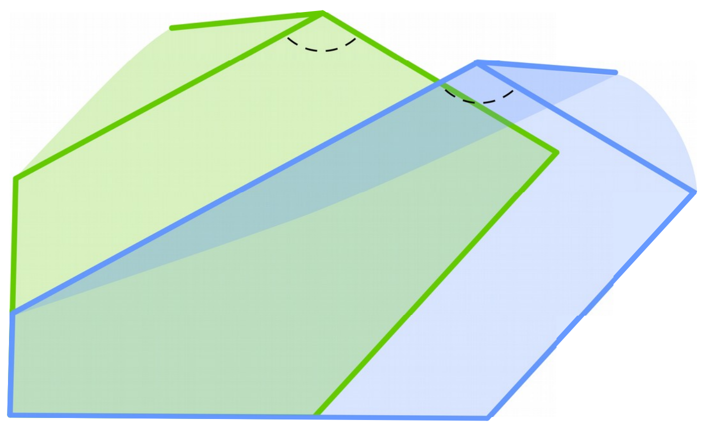
Accidental product – KKL asymptotics

[P. D. and Speziale – To Appear]

The critical point equations:

$$H_a \vec{n}_{ab} = -H_b \vec{n}_{ba}$$

$SL(2,\mathbb{C})$ and closure



[Bahr & collaborators 2016-2017]

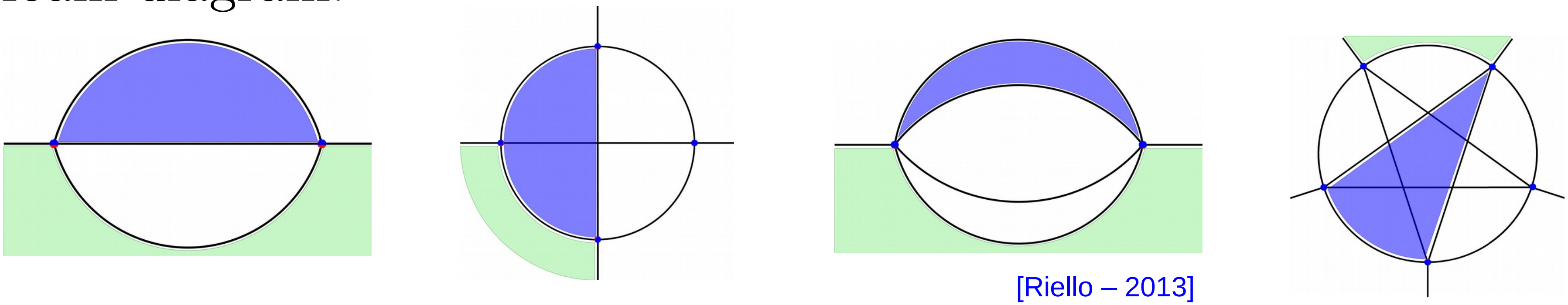
[P.D. , Fanizza, Sarno, Speziale – 2017]

Geometry type	Saddle points
twisted	0
vector *(anti-parallel)*	1
conformal twisted *(angle-matching)*	2
Regge *(shape-matching)*	2

Estimate of infrared divergences

[P.D. – 2018]

Algorithm to bound from above the large volume divergence of any EPRL spin foam diagram.



Main input: numerical scaling of booster functions

Evaluate amplitude numerically shows that the bound is a good estimate

Λ = IR cutoff	bubble 3D	ball 3D	bubble 4D	ball 4D
BF	$\Lambda^{3\mu}$	$\Lambda^{4\mu-1}$	$\Lambda^{10\mu-1}$	$\Lambda^{20\mu-15/2}$
EPRLs	$\Lambda^{3\mu-6}$	$\Lambda^{4\mu-13}$	$\Lambda^{10\mu-13}$	$\Lambda^{20\mu-75/2}$
EPRL	$\Lambda^{3\mu-4}$	$\Lambda^{4\mu-9}$	$\Lambda^{10\mu-1}$	$\Lambda^{20\mu-15/2}$

running of μ

Generalized face weight $A_f(j_f) = (2j_f + 1)^{\mu}$

The flatness problem

[Freidel and Conrady – 2008]
[Bonzom – 2009]
[Hellmann and Kaminski – 2013]

The spin foam partition function in the classical limit is dominated by flat space-time geometries.

Path integral in quantum mechanics

$$\langle x_f\, t_f |\, x_i\, t_i\rangle = \int \mathcal{D}x(t) e^{iS[x(t)]/\hbar}$$

semi-classical limit is dominated by classical trajectories

$$= \int \mathrm{d}x_m \langle x_f\, t_f |\, x_m\, t_m\rangle\langle x_m\, t_m |\, x_i\, t_i\rangle$$

Stationary phase point in $x_m(t_m)$

Spin foam partition function

$$W_\Delta = \sum_{bulk} \prod_f A_f \prod_e A_e \prod_v A_v$$

What does dominate the bulk summation? Argument:

$$A_v \approx \exp\left(i\gamma S_R\right) = \exp\left(i\gamma \sum_f \theta_f j_f\right)$$

$$\delta/\delta j_f \to \gamma\Theta_f = 0 \mod 2\pi$$

Assuming areas independent.

How can we use numerics to study this problem?

Ponzano-Regge model

[Ponzano and Regge - 1969]

Quantum Euclidean gravity in 3D. Perfect sandbox to understand what we are looking for.

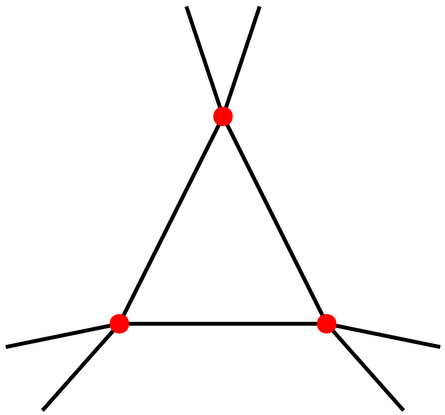
$$A_v = \left\{ \begin{array}{ccc} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{array} \right\}$$

$$A_v \approx \frac{1}{\sqrt{12\pi V}} \cos(S_R + \pi/4)$$

Regge action of a tetrahedron

[Regge - 1968]

Simplest transition amplitude we can compute with any internal face has 3 vertices



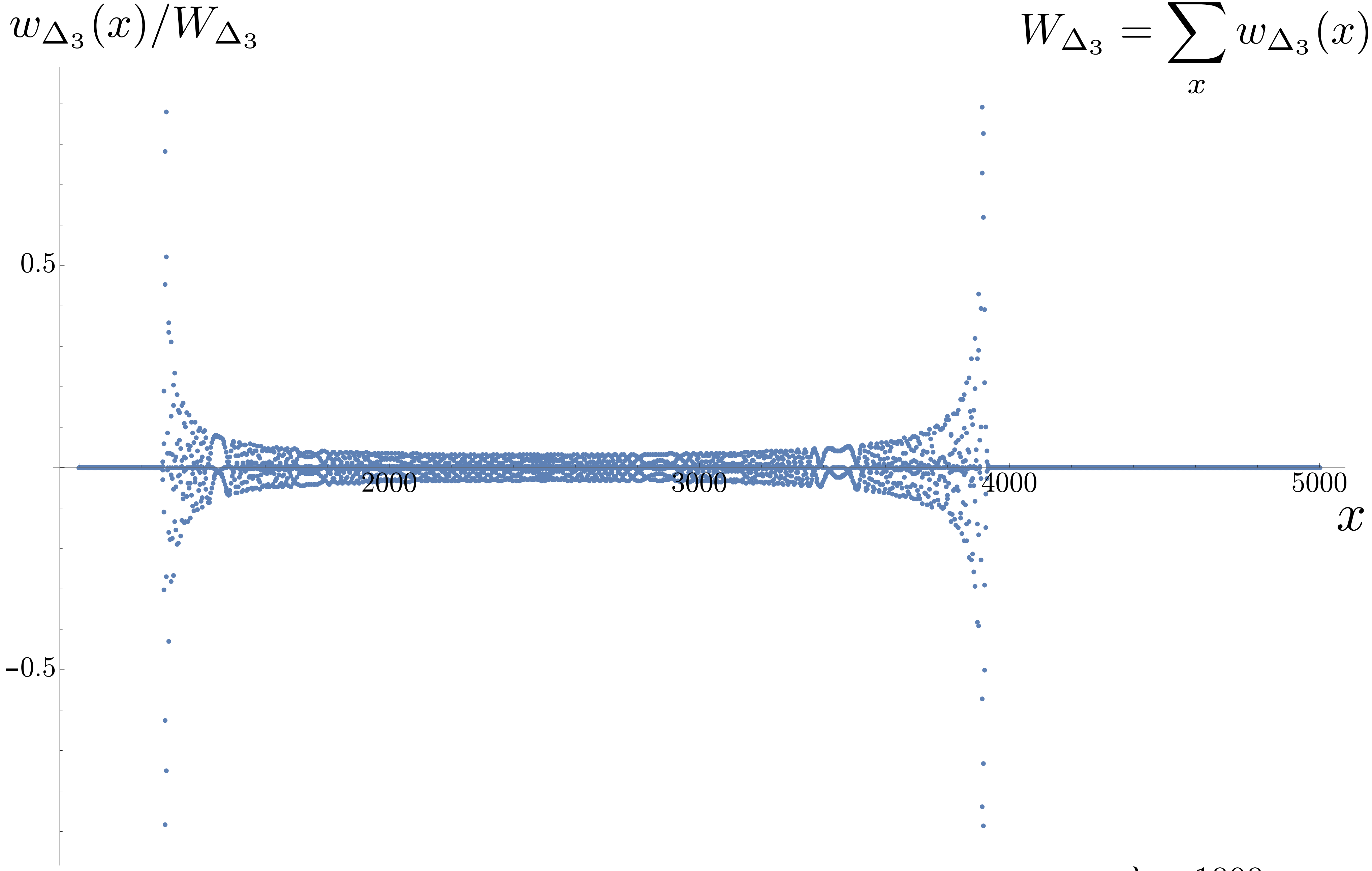
$$W_{\Delta_3} = \sum_x (-1)^x (2x+1) \left\{ \begin{array}{ccc} j_1 & j_5 & j_6 \\ x & j_9 & j_8 \end{array} \right\} \left\{ \begin{array}{ccc} j_6 & j_2 & j_4 \\ j_7 & x & j_9 \end{array} \right\} \left\{ \begin{array}{ccc} j_7 & j_8 & j_3 \\ j_4 & j_5 & x \end{array} \right\}$$

What bulk spin dominate the sum in the large spin regime?

Ponzano-Regge model

[P.D. Sarno and Gozzini – to appear]

$$W_{\Delta_3} = \sum_x w_{\Delta_3}(x)$$

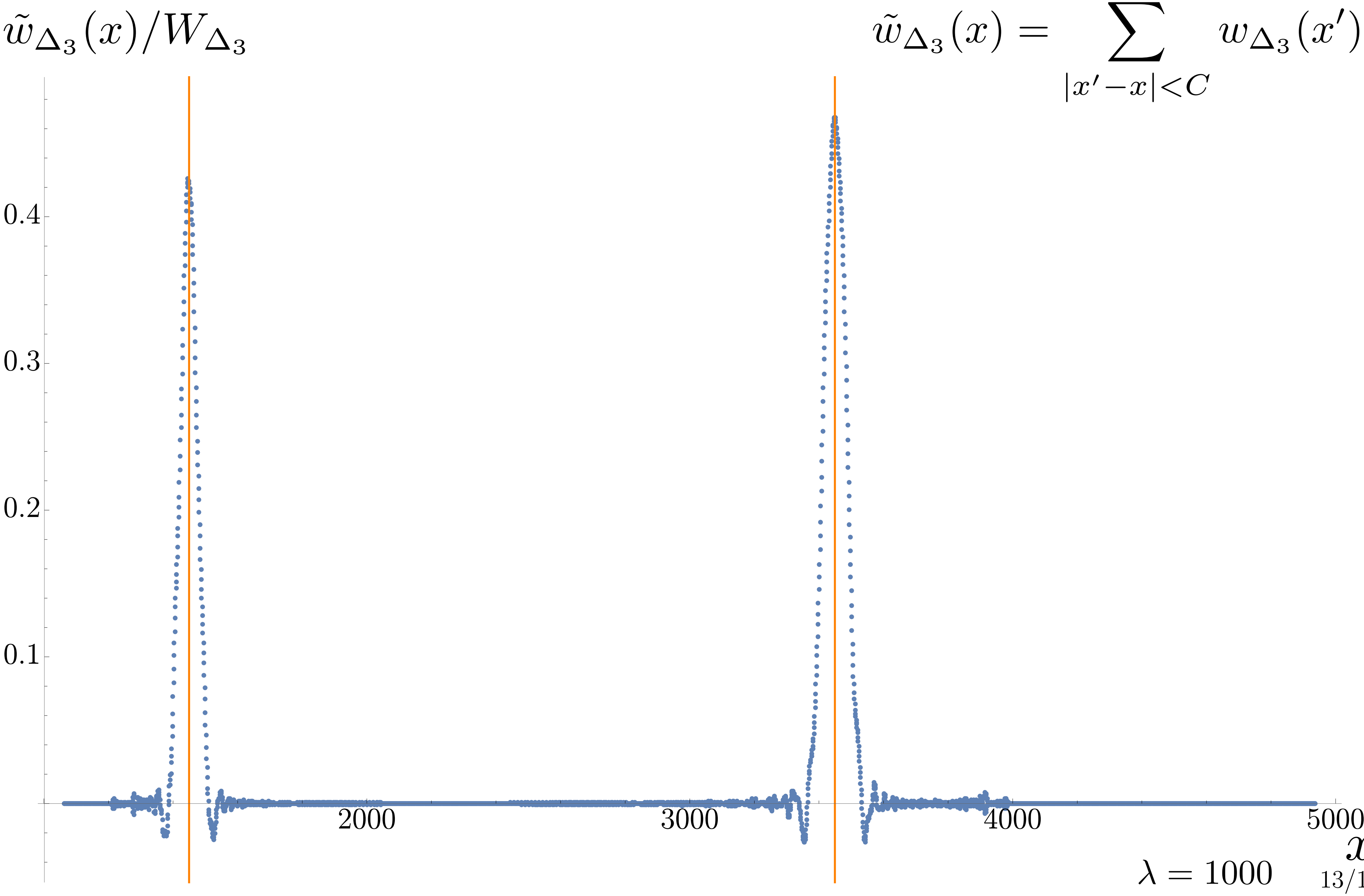


$\lambda = 1000$

Ponzano-Regge model

[P.D. Sarno and Gozzini – to appear]

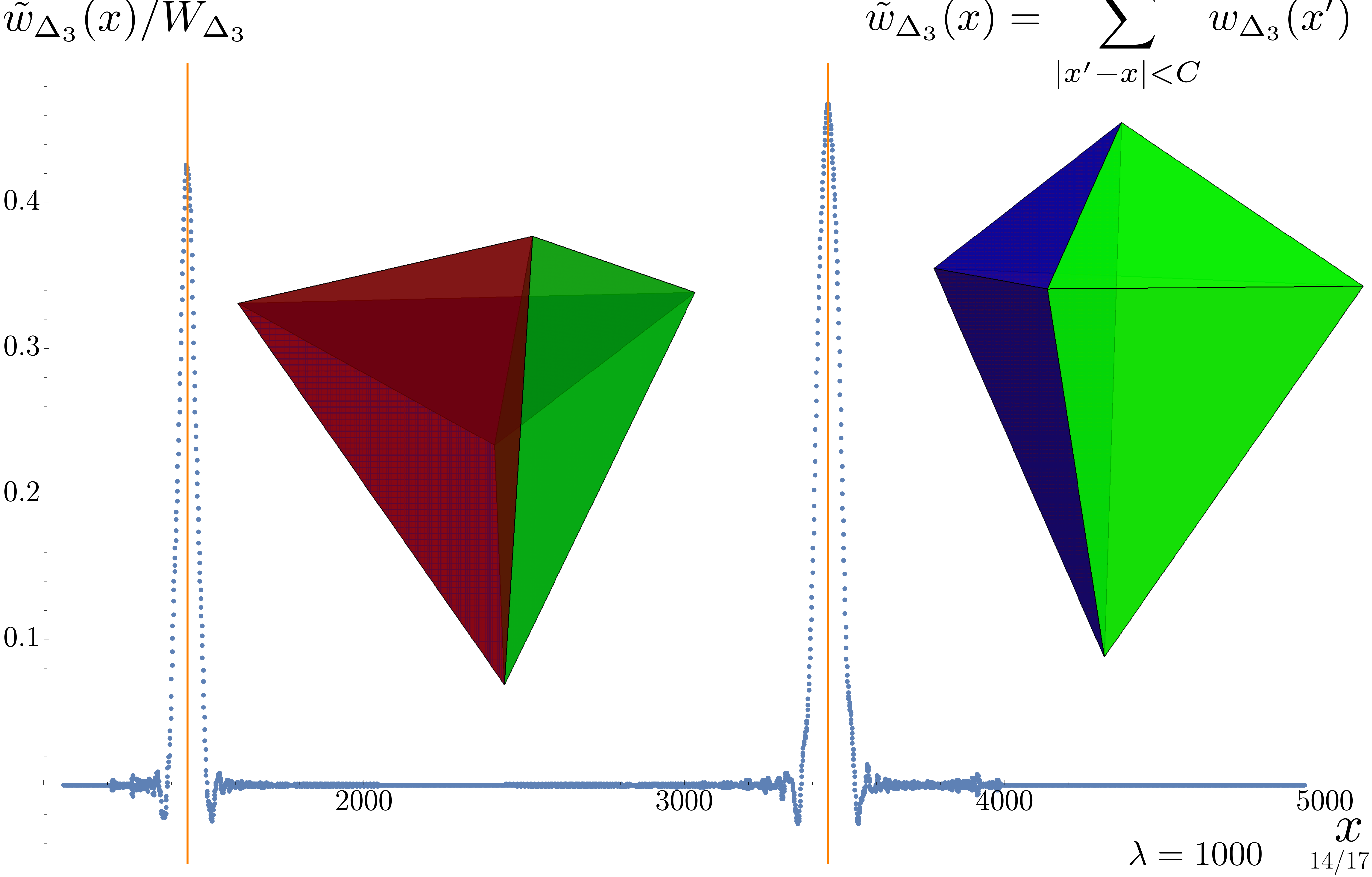
$$\tilde{w}_{\Delta_3}(x) = \sum_{|x'-x|<C} w_{\Delta_3}(x')$$



Ponzano-Regge model

[P.D. Sarno and Gozzini – to appear]

$$\tilde{w}_{\Delta_3}(x) = \sum_{|x'-x|<C} w_{\Delta_3}(x')$$



Ponzano-Regge model

[P.D. Sarno and Gozzini – to appear]

The original Regge argument

$$W_{\Delta_3}(x) \approx \int \mathrm{d}x \exp(iS_R^{(1)}(x) + iS_R^{(2)}(x) + iS_R^{(3)}(x)) + \cdots$$

stationary phase → $\theta_x^{(1)} + \theta_x^{(2)} + \theta_x^{(3)} = 0$

The other exponentials do not interfere.

Cosine problem? No. Cosine feature!

Biedenharn Elliot Identity

$$W_{\Delta_3} = \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_7 & j_8 & j_9 \end{Bmatrix}$$

$$W_{\Delta_3} \approx \cos(S_1 + \frac{\pi}{4}) \cos(S_2 + \frac{\pi}{4}) = \boxed{\sin(S_1 + S_2)} + \boxed{\cos(S_1 - S_2)}$$

Geometry 1 Geometry 2

What did we learn?

Strategy:

Use numerical analysis to hunt for stationary phase points in the bulk summation in the semi-classical limit.

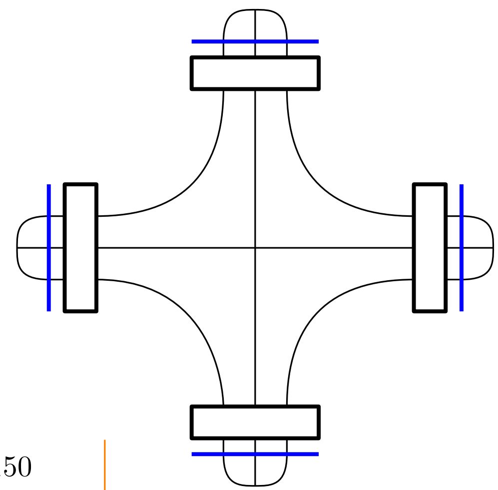
Once we find stationary phase points we translate them in geometries.

We try to deduct information on the classical theory.

Ponzano Regge as a proof of concept for numerics.

Cosine instead of exponential in the asymptotic is not a problem

Cosine asymptotic helps generating all possible geometries



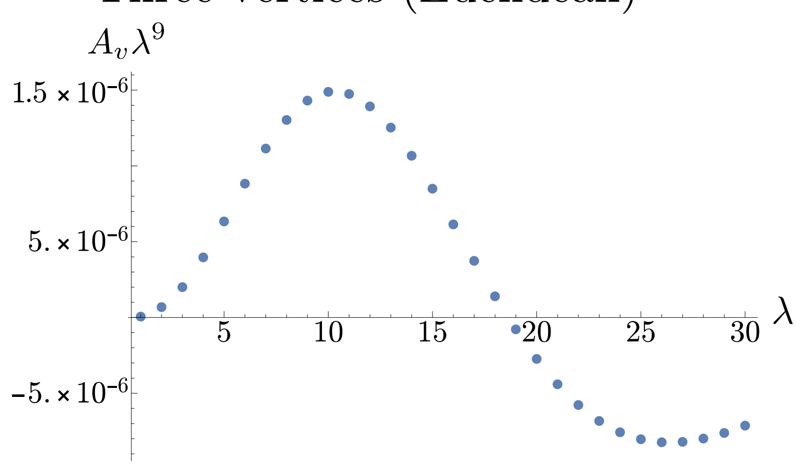
Three dimensional version of the EPRL model

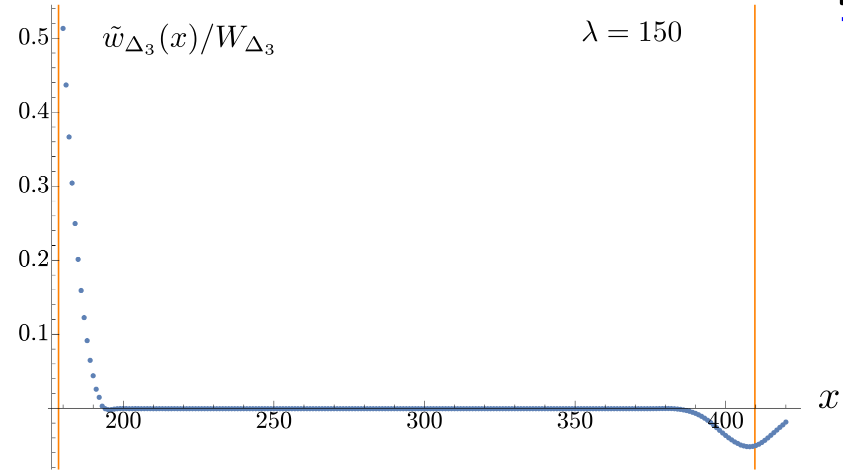
Symmetry group is SL(2,C)

Compare with Lorentzian Ponzano Regge [Freidel – 2001]

Single vertex (Euclidean, Lorentzian)

Three vertices (Euclidean)





What did we learn?

Strategy:

Use numerical analysis to hunt for stationary phase points in the bulk summation in the semi-classical limit.

Once we find stationary phase points we translate them in geometries.

We try to deduct information on the classical theory.

Ponzano Regge as a proof of concept for numerics.

Cosine instead of exponential in the asymptotic is not a problem

Cosine asymptotic helps generating all possible geometries

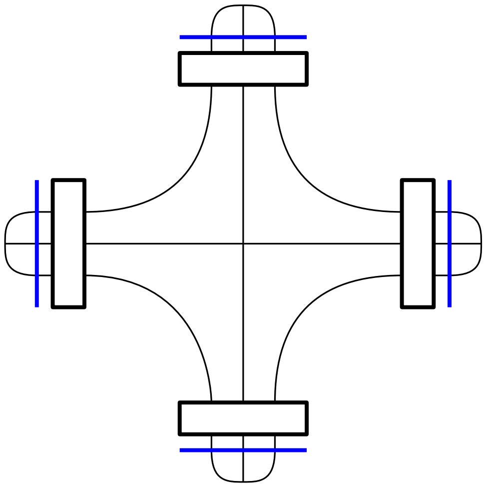
Three dimensional version of the EPRL model

Symmetry group is SL(2,C)

Compare with Lorentzian Ponzano Regge [Freidel – 2001]

Single vertex (Euclidean, Lorentzian)

Three vertices (Euclidean)



Full EPRL model in 4D.

Preliminary results in the Euclidean model [Bayle, Collet, Rovelli – 2016] [J.R. Oliveira - 2017]

Full Lorentzian model is at our doorstep

Set up boundary data compatible with curved bulk and observe the presence (or absence) of stationary phase points in the sum over bulk variables

Summary and Conclusion

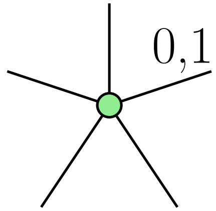
Analytical computation of a spin foam transition amplitude in the EPRL model is very complicated.

Precise numerical evaluations are possible and public

On GitHub and also on the "Encyclopedia of quantum geometries"

Reproduce semi-classical expressions, ready for the exploration of the deep quantum regime

KKL model large spin asymptotic



Very important tool in the study of IR divergences

Scaling of the booster functions. Check: the bound from above is a good estimate

Strategy to solve the flatness problem

Look for boundary data corresponding to curved geometries

Connections with real world: compute transition amplitude of quantum physical processes.

First application to cosmology is out [Gozzini and Vidotto 2019]

Thank you for your attention