Spin Foam: a numerical revolution

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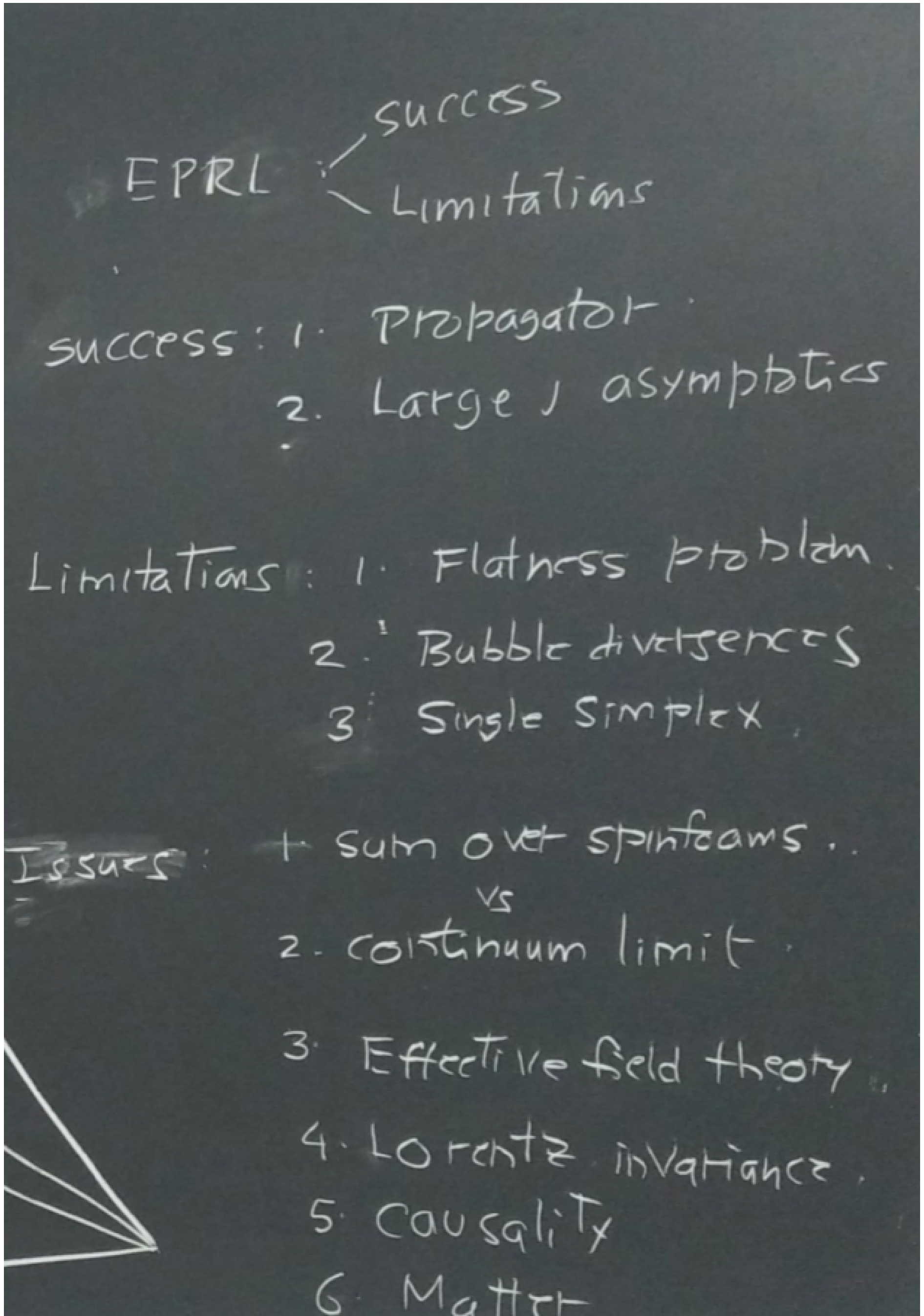
arXiv:1708.01727 arXiv:1803.00835

Motivations



15th December 2017 - Informal board discussion on Spin Foams

Covariant formulation of LQG dynamics

State of art: EPRL-FK model

Extremely complex computations

Alesci, Bianchi, Magliaro, Perini, Rovelli, Zhang

Barret, Gomes, Hellmann et al.

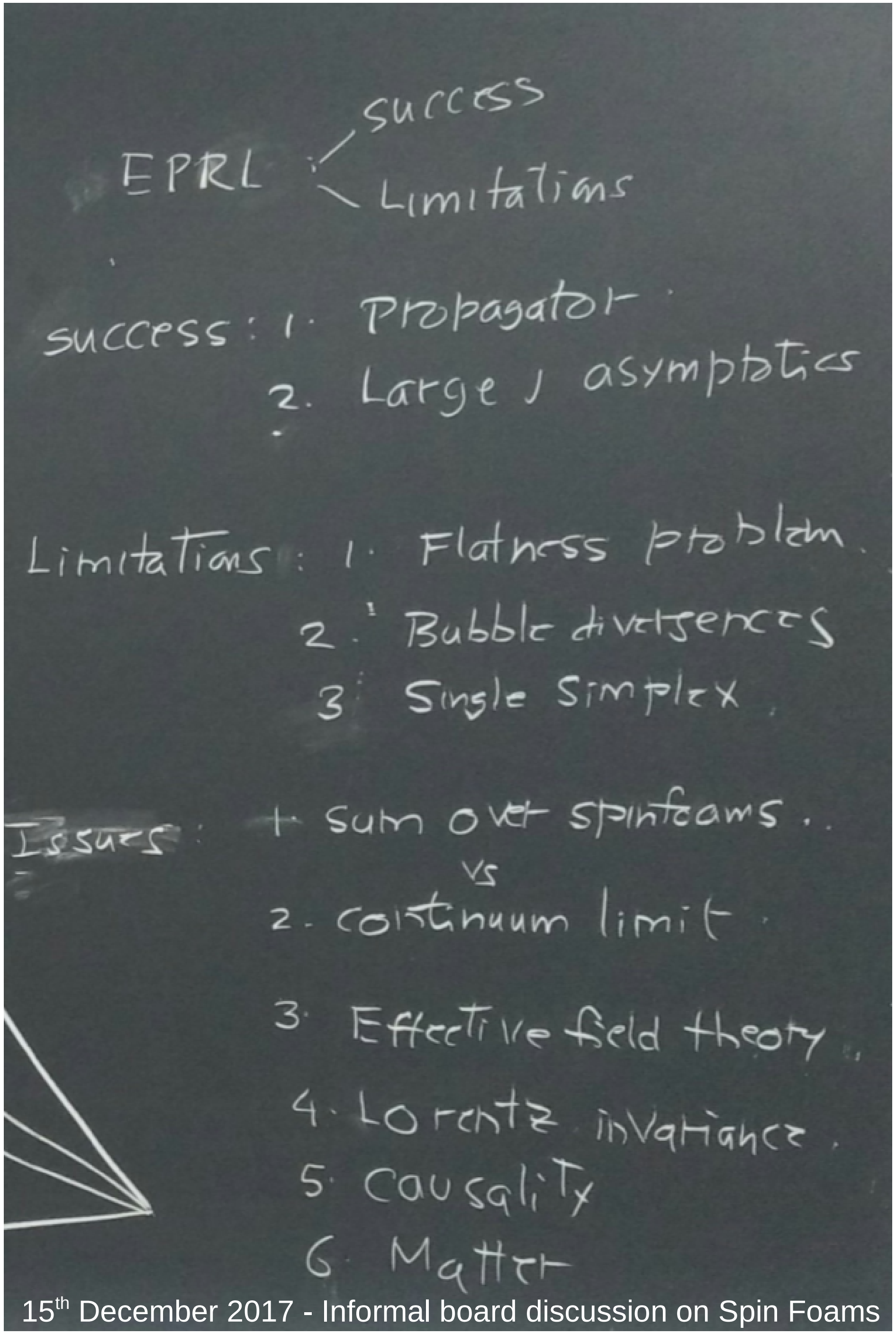
Freidel, Conrady, Bonzom, Hellmann, Kaminski

Bonzom, Perini, Rovelli, Riello, Smerlak, Speziale

Smerlak, Rovelli

Bahr, Delcamp, Dittrich, Geiller, Steinhaus

Motivations



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Covariant formulation of LQG dynamics

State of art: EPRL-FK model

Extremely complex computations

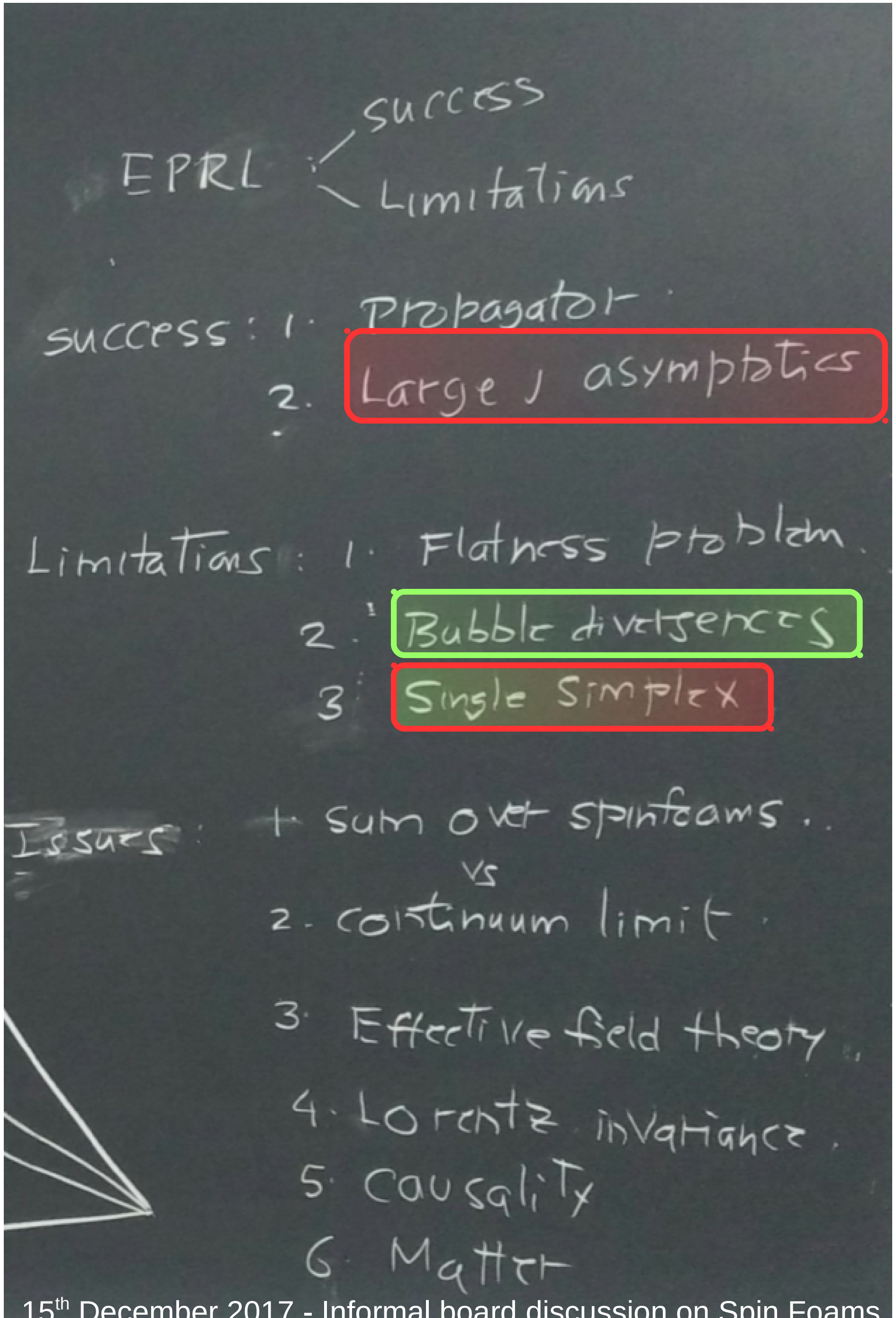
Numerical Evaluation of Spin Foam transition amplitudes is within our reach

What can we learn from it?

Disclaimer: despite what my title suggests, the path to complete the program is still long.

Any input is very welcome!

Motivations



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What can we learn from it?

Today

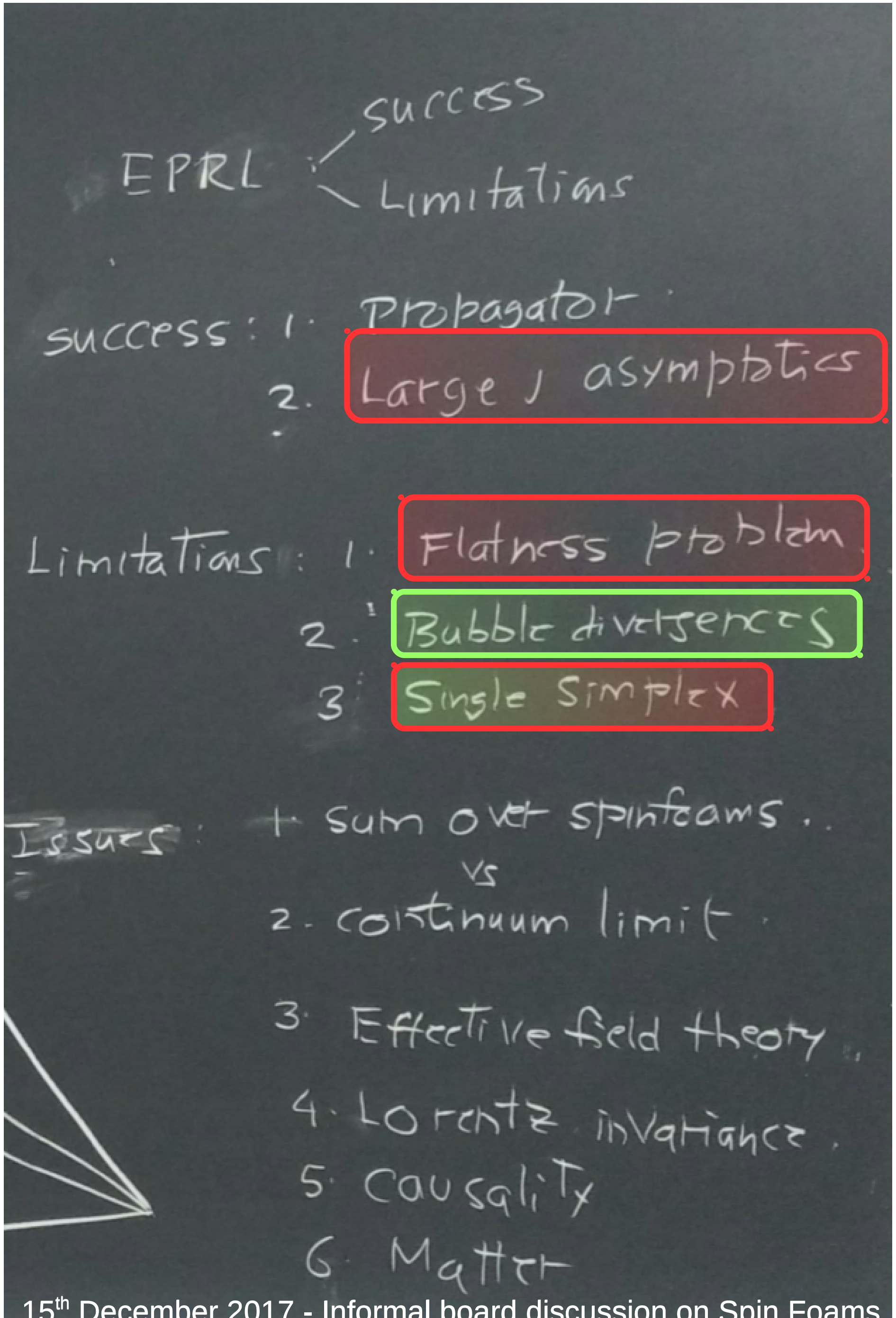
2017 – P.D., Fanizza, Sarno and Speziale

In Prep. – P.D., Fanizza, Sarno and Speziale

2018 – Sarno, Stagno and Speziale

2018 – P.D.

Motivations



15th December 2017 - Informal board discussion on Spin Foams

Covariant formulation of LQG dynamics

State of art: EPRL-FK model

Extremely complex computations

Numerical Evaluation of Spin Foam transition amplitudes is within our reach

What can we learn from it?

Today

Tomorrow

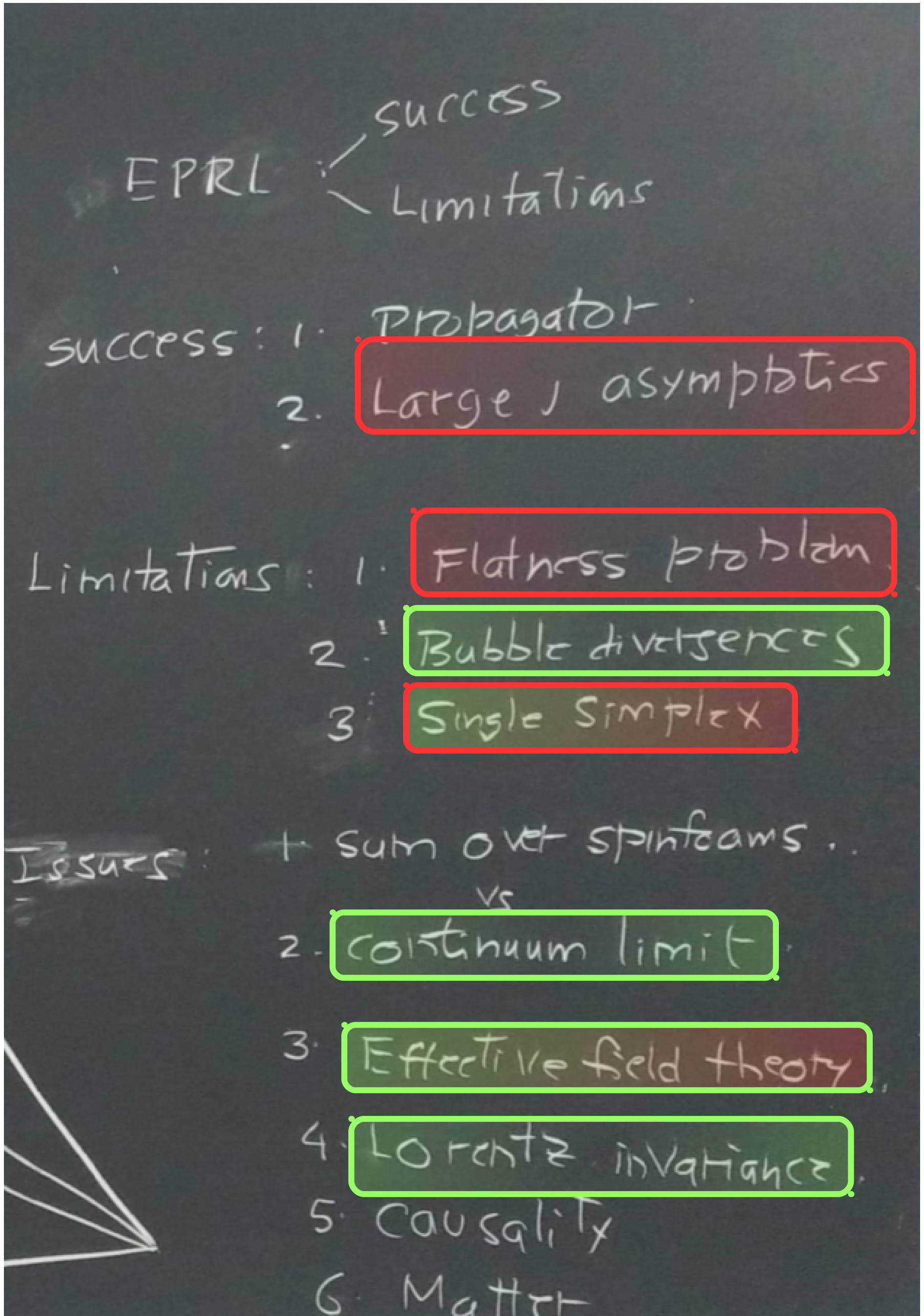
Many simplicies:

Two Vertices – Bubbles (4D, tensorial structure)

Three Vertices – Delta 3 (flatness?)

P.D., Sarno, Collet, Speziale

Motivations



15th December 2017 - Informal board discussion on Spin Foams

Covariant formulation of LQG dynamics

State of art: EPRL-FK model

Extremely complex computations

Numerical Evaluation of Spin Foam transition amplitudes is within our reach

What can we learn from it?

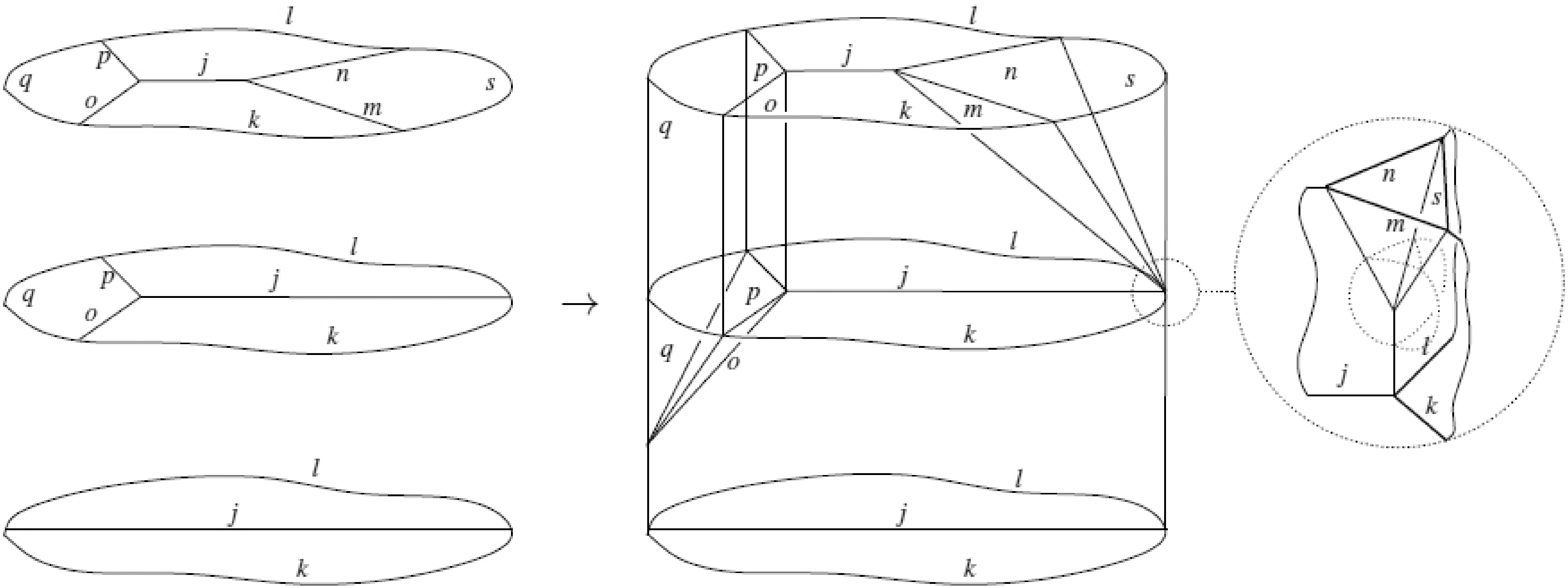
Today

Tomorrow

The day after
- Wilsonian RG flow
- Tensor network
- Recover Lorentz invariance at FP
- Connection with Perturbative QG
- Phenomenology

Spin Foams: partition function

$$Z_{\mathcal{C}} = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f,\ i_e)$$

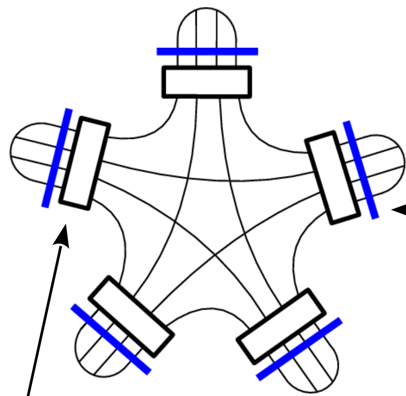


[2013 – Living review – Perez]

Spin Foams: EPRL-FK model

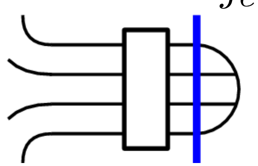
$$Z_{\mathcal{C}} = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f,\ i_e)$$

$A_v(j_f, i_e) =$



Y map : $j \longrightarrow (\rho, k) \overset{Y}{=} (\gamma j, j)$

$SL(2, \mathbb{C})$



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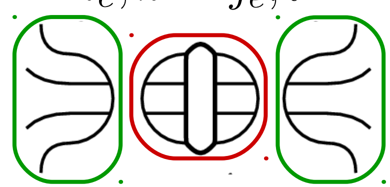


$B_4(j_f, i_e, l_{fv}, k_{ev}; \gamma)$

Cartan Decomposition: $u e^{\frac{r}{2}\sigma_3} v^{-1}$

[2016 – Speziale]

Booster functions &

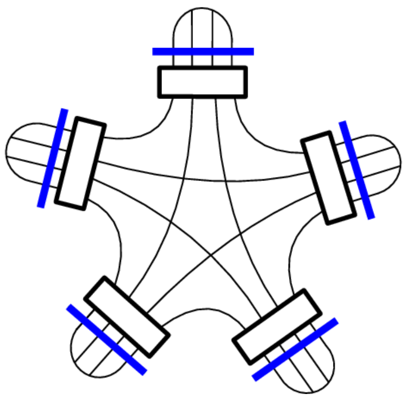
SU(2) Intertwiners

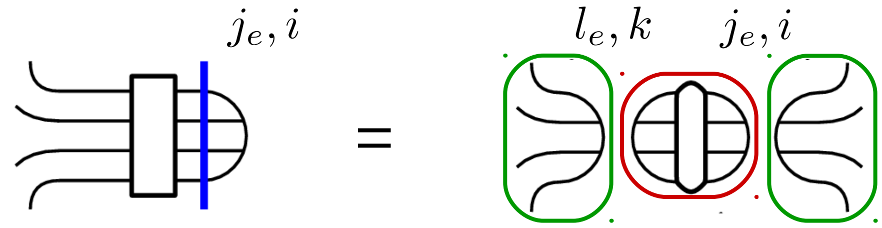
Spin Foams: EPRL-FK model

$$Z_{\mathcal{C}} = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v\left(j_f,\ i_e\right)$$

[2016 – Speziale]

$A_v\left(j_f, i_e\right) =$



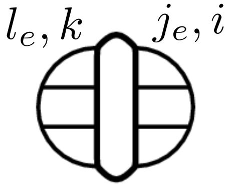


$B_4\left(j_f, i_e, l_{fv}, k_{ev}; \gamma\right)$

Take home message:
We can decompose the EPRL-FK vertex amplitude into a superposition of SU(2) invariants weighted by Boosters functions (one per half-edge)

$$A_v\left(j_f,\ i_e\right) = \sum_{l_{fv}, k_{ev}} \left(\prod_{ev} \left(2k_{ev}+1\right) B_4(j_{fv}, l_{fv}; i_{ev}, k_{ev}) \right) \{15j\}_v(l_{fv}, k_{ev})$$

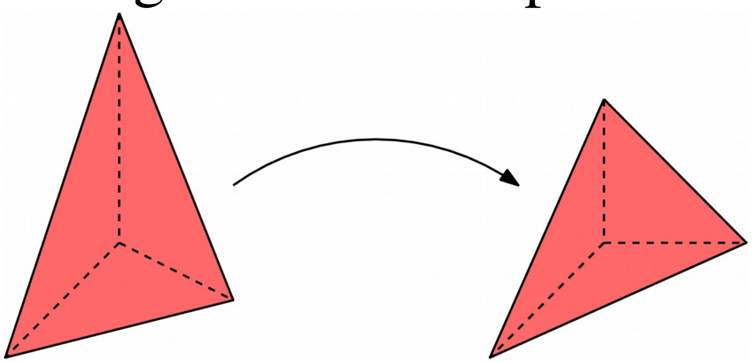
The Booster Functions



$$B_n(j_e, l_e; i, k) = \frac{1}{4\pi} \sum_{p_e} \begin{pmatrix} j_e \\ p_e \end{pmatrix}^{(i)} \left(\int_0^\infty \mathrm{d}r \sinh^2 r \prod_{e=1}^{n} d^{(\gamma j_e, j_e)}_{j_e l_e p_e}(r) \right) \begin{pmatrix} l_e \\ p_e \end{pmatrix}^{(k)}$$

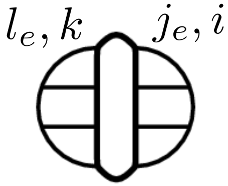
Intriguing asymptotic and

geometric interpretation



not the end of the story, more work is needed

The Booster Functions



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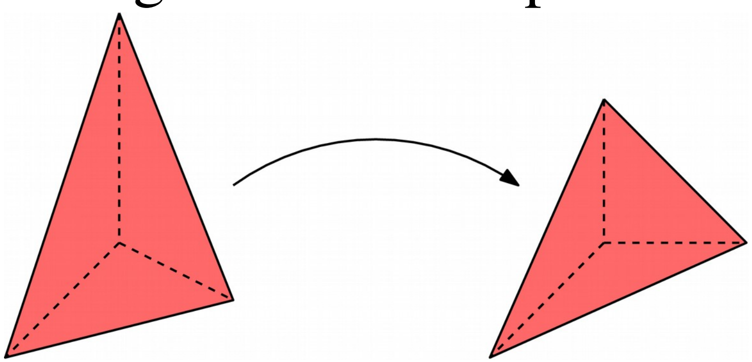


not the end of the story, more work is needed

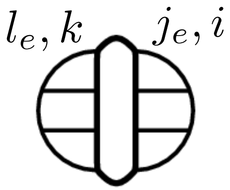
Numerical calculability:

In terms of SL(2,C) Clebsch–Gordan coefficients (finite sums of ratios of Gamma functions)

Brute-force integration of the rapidity integrals (after some manipulations)

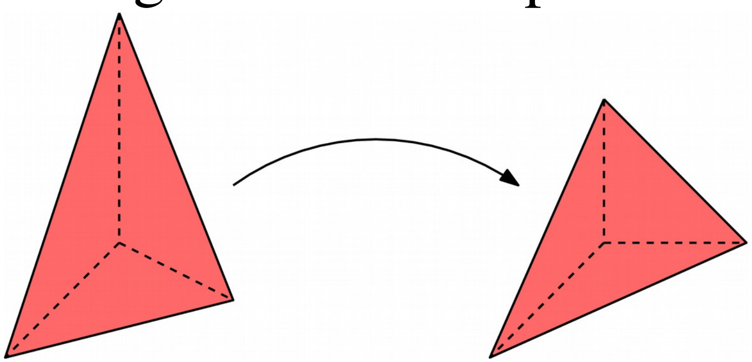
Time and precision:
arbitrary precision mathematics (interference)
is HPC necessary? (parallelization)

The Booster Functions



$$B_n(j_e, l_e; i, k) = \frac{1}{4\pi} \sum_{p_e} \begin{pmatrix} j_e \\ p_e \end{pmatrix}^{(i)} \left(\int_0^\infty \mathrm{d}r \sinh^2 r \prod_{e=1}^{n} d^{(\gamma j_e, j_e)}_{j_e l_e p_e}(r) \right) \begin{pmatrix} l_e \\ p_e \end{pmatrix}^{(k)}$$

Intriguing asymptotic and geometric interpretation



not the end of the story, more work is needed

Numerical calculability:

In terms of SL(2,C) Clebsch–Gordan coefficients (finite sums of ratios of Gamma functions)

Brute-force integration of the rapidity integrals (after some manipulations)

Time and precision:
arbitrary precision mathematics (interference)
is HPC necessary? (parallelization)

Our current code is written in C, integrates and use arbitrary precision math libraries. We can compute Booster functions with spins of order 50 in minutes

credits to the many students that have been sacrificed for this cause – Sarno & Collet

Back to the Vertex Amplitude

$$A_v\left(j_f, i_e\right) = \sum_{l_{fv}, k_{ev}} \left(\prod_{ev} \left(2k_{ev}+1\right) B_4^{\checkmark}(j_{fv}, l_{fv}; i_{ev}, k_{ev}) \right) \boxed{\{15j\}_v(l_{fv}, k_{ev})}$$

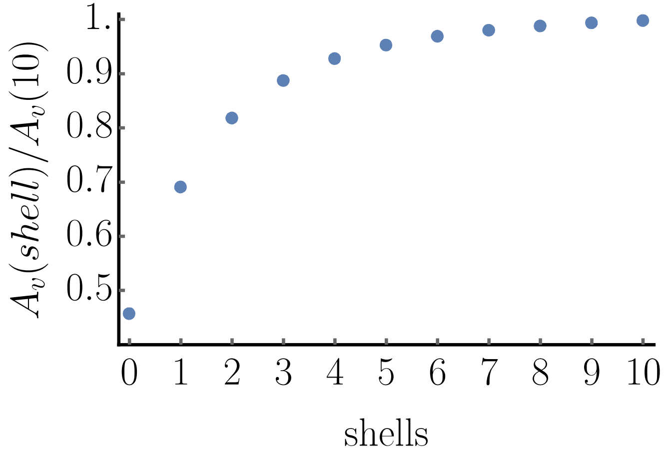
Computing SU(2) invariants is not easy

- dedicated algorithm for 3j, 6j and 9j symbols [2015 - Johansson, Forssén]
- take into account symmetries to save memory and time
- smart basis choice leads to reducible symbols
- as a warm up exercise we checked the $\{15j\}$ symbol asymptotic

Back to the Vertex Amplitude

$$A_v\left(j_f, i_e\right) = \sum_{l_{fv}, k_{ev}} \left(\prod_{ev} (2k_{ev}+1)\, B_4(j_{fv}, l_{fv}; i_{ev}, k_{ev}) \right) \{15j\}_v(l_{fv}, k_{ev})$$

Unbounded but convergent sums! We studied the convergence in shells



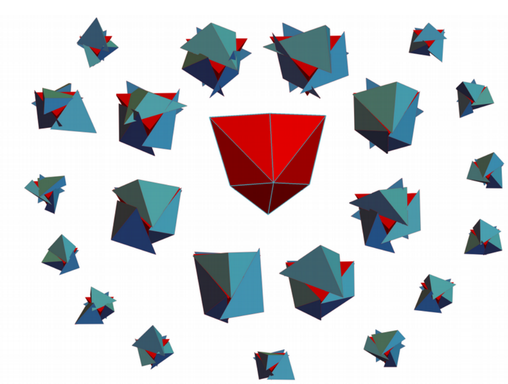
Selection rules are needed. A small percentage of addends really matters

Empirical selection – very rough but effective

Using geometrical intuition coming from the asymptotic

Machine learning

Discrete version of Monte Carlo



Simplifications

Full EPRL-FK Amplitude:

$$A_v\left(j_f,\, i_e\right) = \sum_{l_{fv}, k_{ev}} \left(\prod_{ev} \left(2k_{ev}+1\right) B_4(j_{fv}, l_{fv}; i_{ev}, k_{ev}) \right) \{15j\}_v(l_{fv}, k_{ev})$$

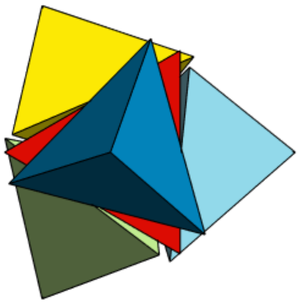
Simplified Model (EPRLs): (extra enforcement of the Y map)

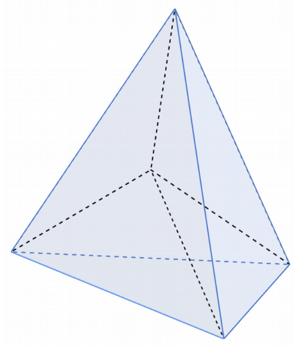
$$A_v\left(j_f,\, i_e\right) = \sum_{\cancel{l_{fv}}, k_{ev}} \left(\prod_{ev} \left(2k_{ev}+1\right) B_4(j_{fv}, l_{fv}; i_{ev}, k_{ev}) \right) \{15j\}_v(l_{fv}, k_{ev})$$

Topological BF SU(2):

$$A_v\left(j_f,\, i_e\right) = \sum_{\cancel{l_{fv}}, k_{ev}} \left(\prod_{ev} \underbrace{\cancel{\left(2k_{ev}+1\right) B_4(j_{fv}, l_{fv}; i_{ev}, k_{ev})}}_{\delta_{k_e i_e}} \right) \{15j\}_v(l_{fv}, k_{ev})$$

Asymptotic of the Vertex Amplitude





What?

Single vertex

Boundary intertwiners coherent states

Uniform scaling

Why?

Simplest amplitude

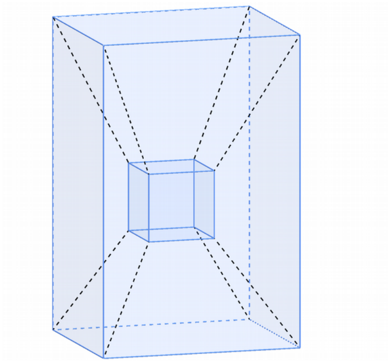
Analytic formulas available (saddle point)

Test drive of the machinery.

How?

One (complexity) step at a time

Various models and boundary geometries

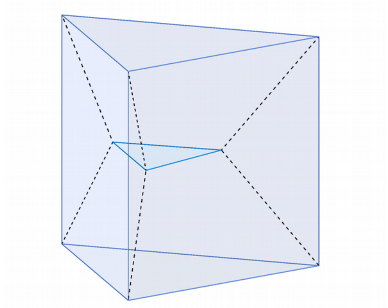


Learn?

Computing Hessians is hard

Cosine "problem"

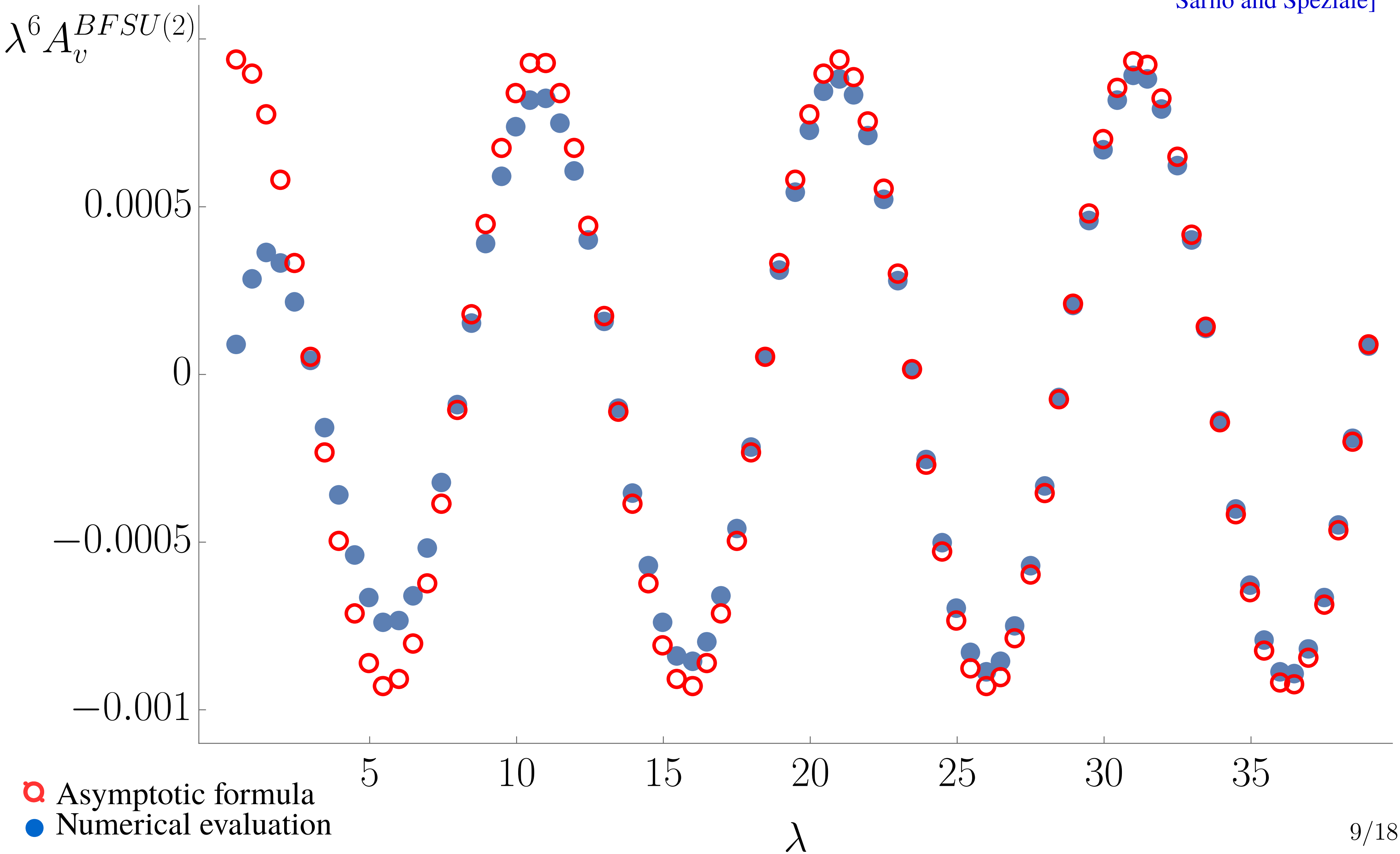
Generalization is possible (KKL)



SU(2) BF Vertex Amplitude

Equilateral 4simplex

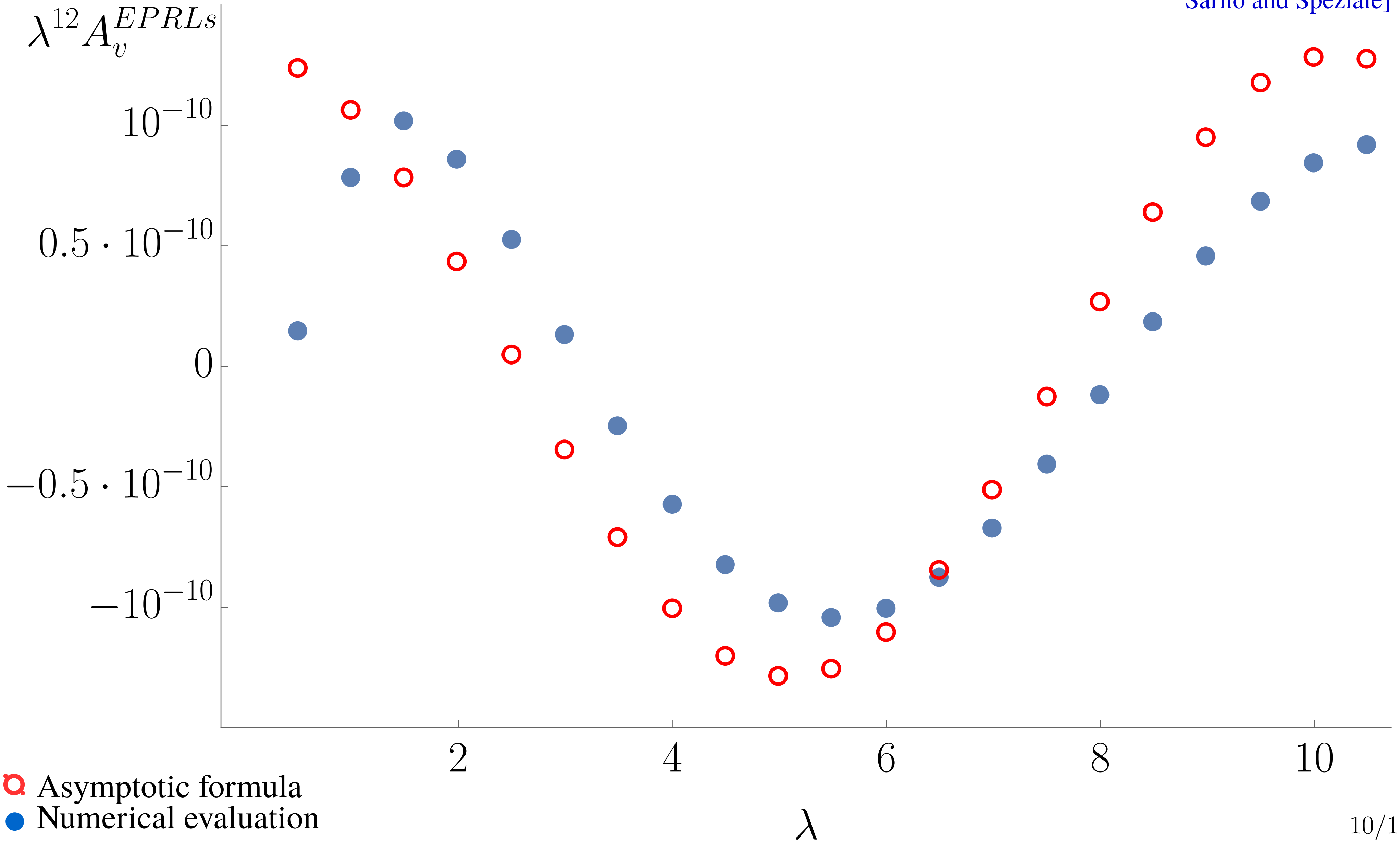
[2017 – P.D., Fanizza, Sarno and Speziale]



EPRLs Vertex Amplitude

0 shell. Semiclassical limit of EPRLs?

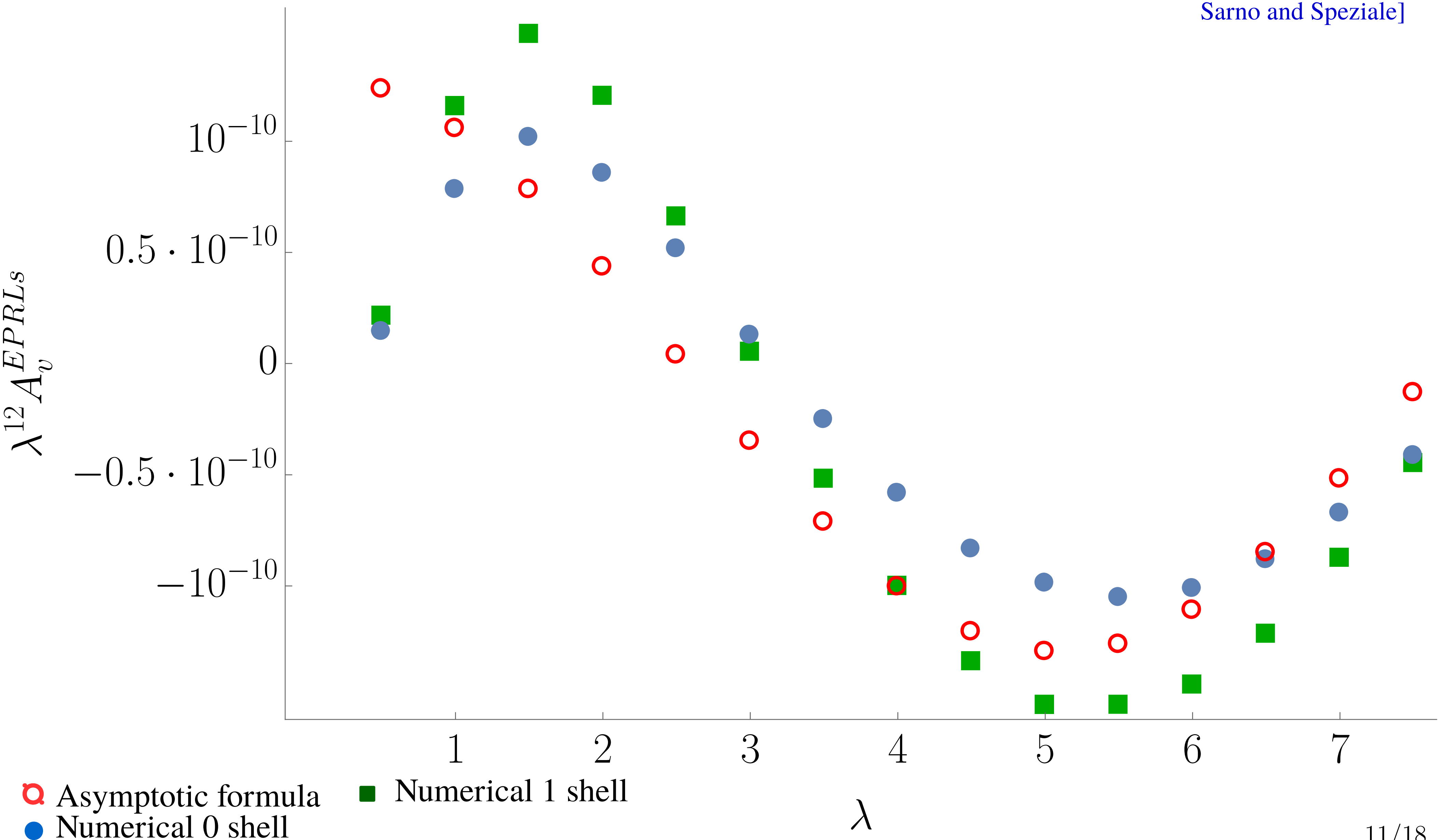
[2017 – P.D., Fanizza, Sarno and Speziale]



EPRL Vertex Amplitude

1 shell. Semiclassical limit of EPRLs?

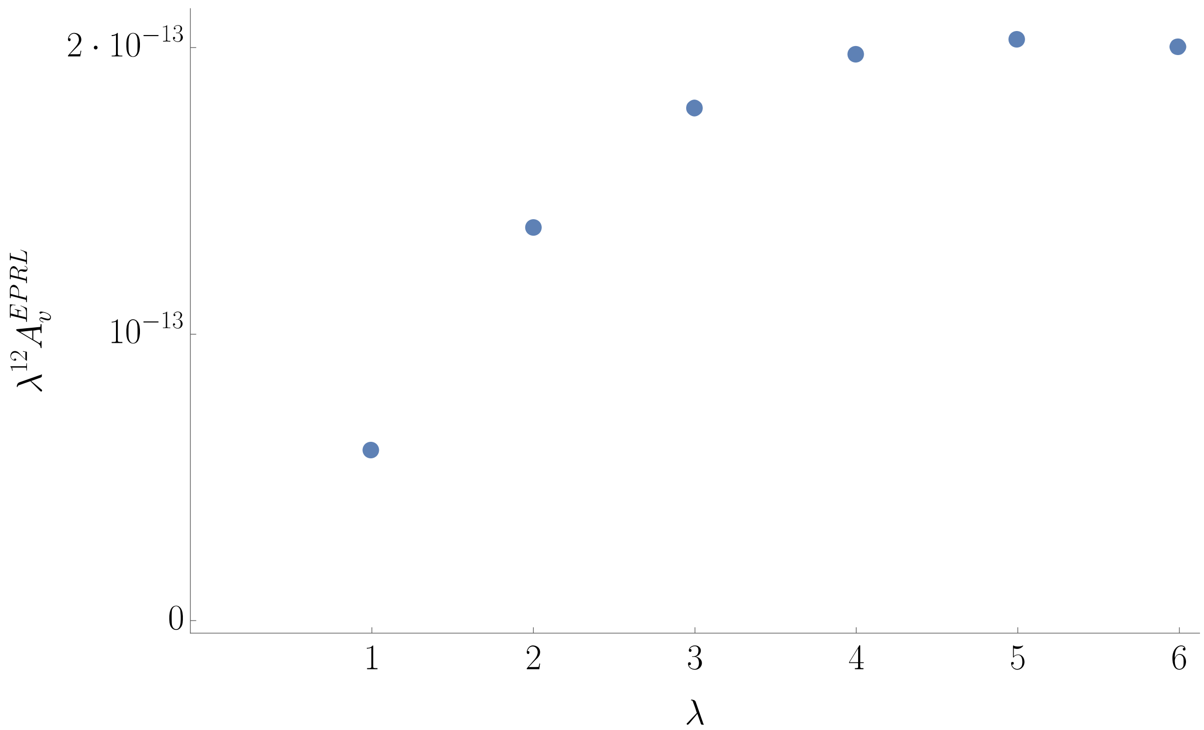
[2017 – P.D., Fanizza, Sarno and Speziale]

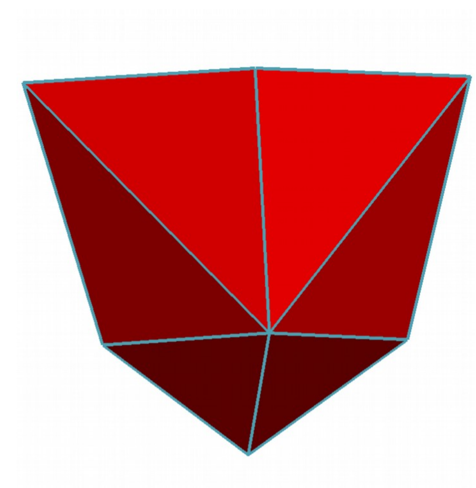


EPRL Vertex Amplitude

Lorentzian Geometry boundary data.

[2017 – P.D., Fanizza, Sarno and Speziale]





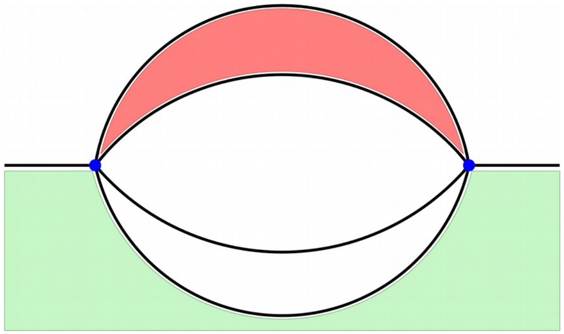
The real limitation is in the boundary data:

- Lorentzian 4simplex
- Boundary made of 5 space-like tetrahedra
- Integer areas (boosted to a common R^3) as similar as possible

} Spins grows too quickly.

Infrared divergences

Bubble: collection of faces in the cellular complex forming a closed 2-surface



$$W = \sum_{j_f, i_e} \prod_f (2j_f + 1)^\mu \prod_e (2i_e + 1) \prod_v A_v(j_f,\ i_e)$$

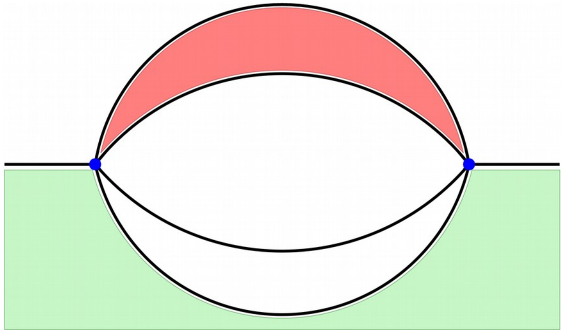
Problem studied in the literature

Euclidean SO(4) - [2009 Perini, Speziale, Rovelli]

Lorentzian EPRL (Log Divergence, geometric picture, saddle point) – [2014 Riello]

Infrared divergences

Bubble: collection of faces in the cellular complex forming a closed 2-surface



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Problem studied in the literature

Euclidean SO(4) - [2009 Perini, Speziale, Rovelli]

Lorentzian EPRL (Log Divergence, geometric picture, saddle point) – [2014 Riello]

Algorithm applicable to any diagram

[2018 - P.D.]

Assumptions:

uniform scaling of all the face spins

no interference (estimate from above)

Ingredients:

behavior of SU(2) invariants

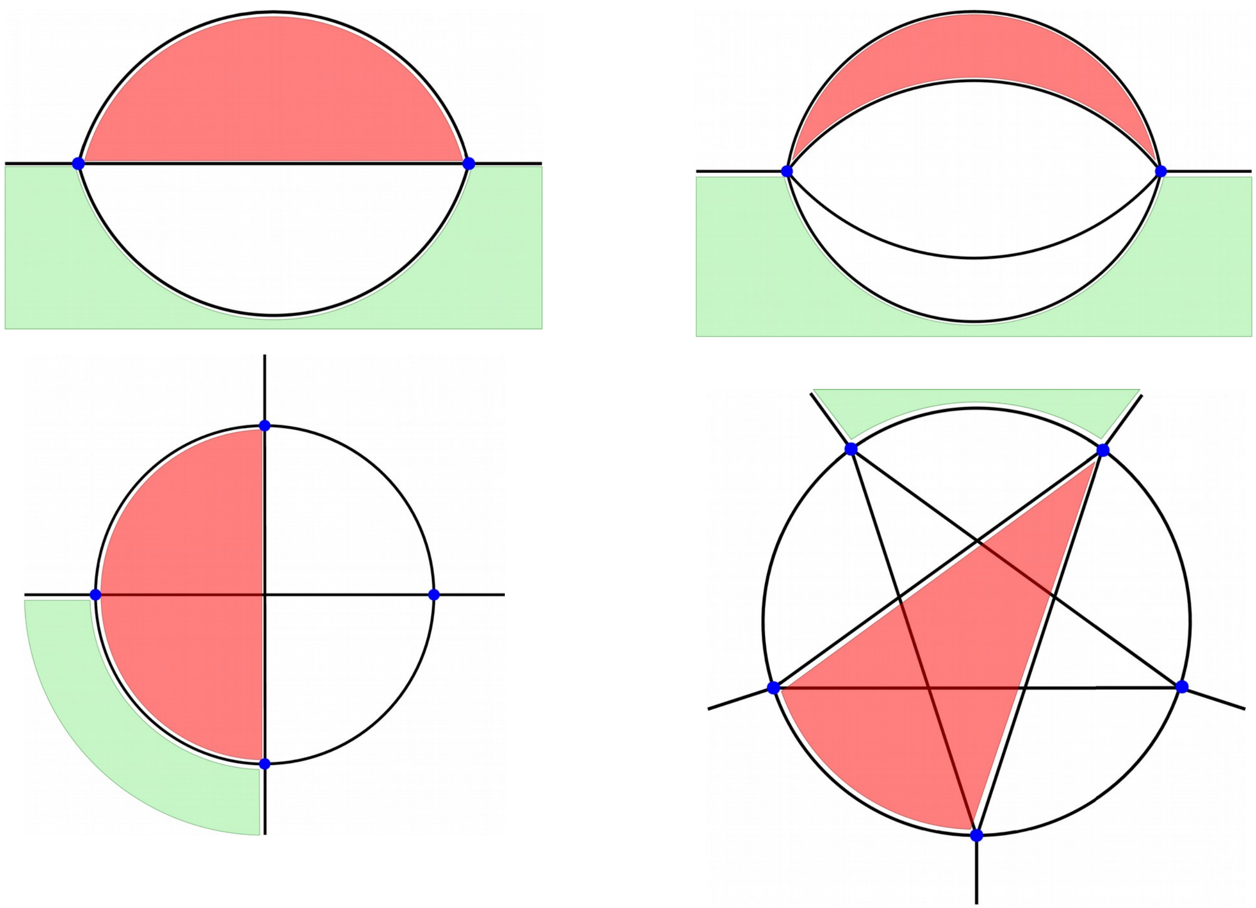
behavior of boosters inferred from numerics

numerical evaluation for simple diagrams

Infrared divergences

[2018 - P.D.]

	bubble 3D	ball 3D	bubble 4D	ball 4D
BF	$\Lambda^{3\mu}$	$\Lambda^{4\mu-1}$	$\Lambda^{10\mu-1}$	$\Lambda^{20\mu-15/2}$
EPRLs	$\Lambda^{3\mu-6}$	$\Lambda^{4\mu-13}$	$\Lambda^{10\mu-13}$	$\Lambda^{20\mu-75/2}$
EPRL	$\Lambda^{3\mu-4}$	$\Lambda^{4\mu-9}$	$\Lambda^{10\mu-1}$	$\Lambda^{20\mu-15/2}$



Infrared divergences

	bubble 3D	ball 3D	bubble 4D	ball 4D
BF	$\Lambda^{3\mu}$	$\Lambda^{4\mu-1}$	$\Lambda^{10\mu-1}$	$\Lambda^{20\mu-15/2}$
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EPRL	$\Lambda^{3\mu-4}$	$\Lambda^{4\mu-9}$	$\Lambda^{10\mu-1}$	$\Lambda^{20\mu-15/2}$

I can evaluate the amplitude analytically and compare

I can evaluate the amplitude numerically and compare

We are working on the numeric for the rest

Infrared divergences

	bubble 3D	ball 3D	bubble 4D	ball 4D
BF	$\Lambda^{3\mu}$	$\Lambda^{4\mu-1}$	$\Lambda^{10\mu-1}$	$\Lambda^{20\mu-15/2}$
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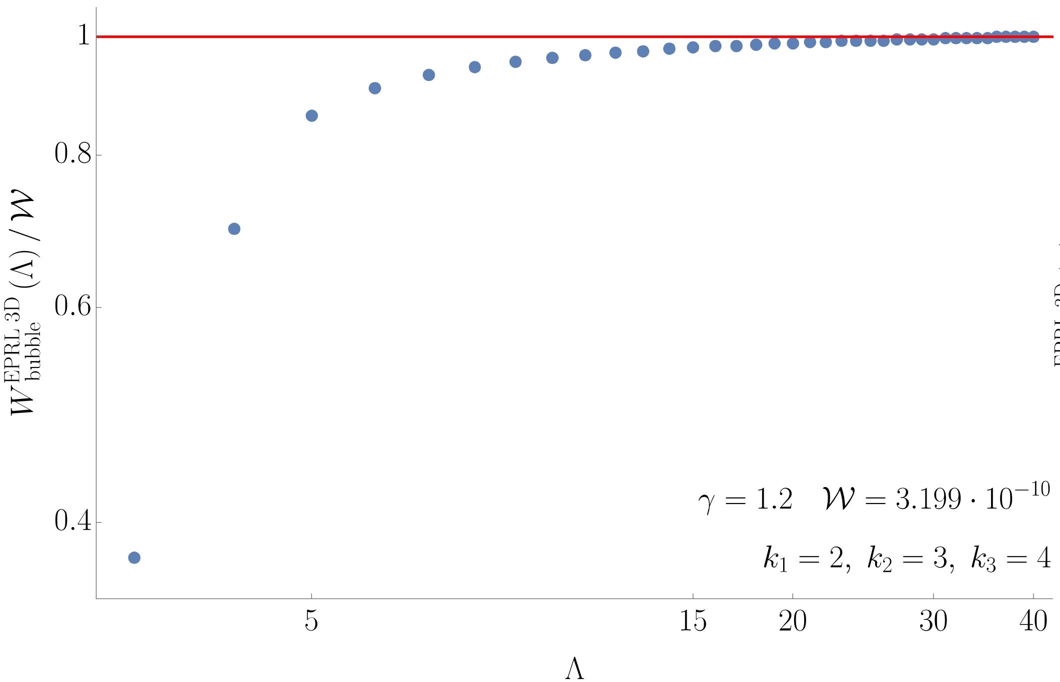
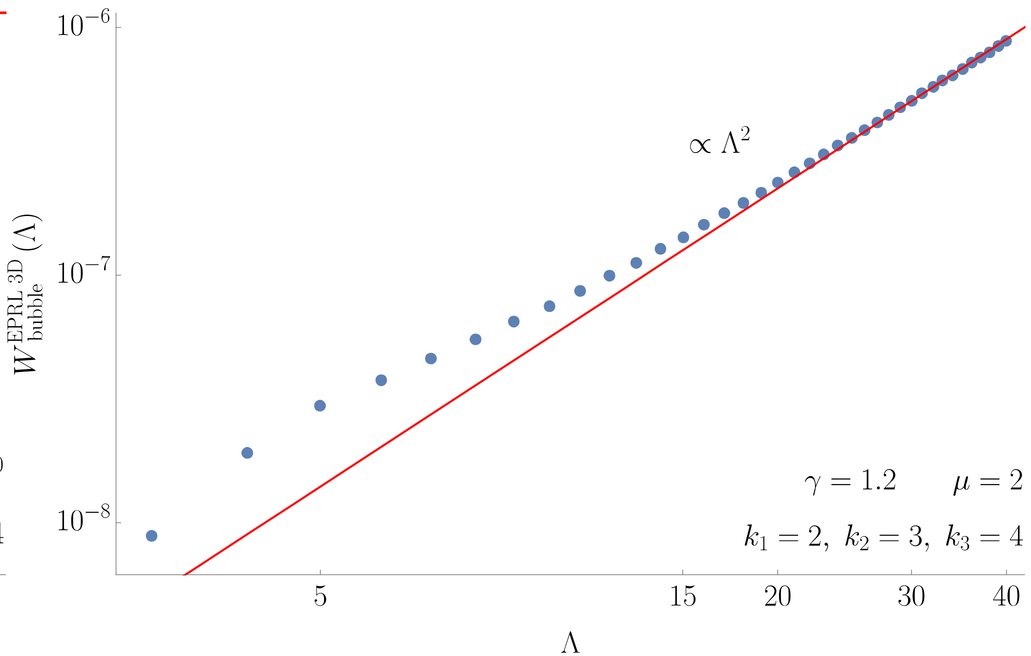
EPRL	$\Lambda^{3\mu-4}$	$\Lambda^{4\mu-9}$	$\Lambda^{10\mu-1}$	$\Lambda^{20\mu-15/2}$



$\mu = 1$



$\mu = 2$

Infrared divergences

	bubble 3D	ball 3D	bubble 4D	ball 4D
BF	$\Lambda^{3\mu}$	$\Lambda^{4\mu-1}$	$\Lambda^{10\mu-1}$	$\Lambda^{20\mu-15/2}$
EPRLs	$\Lambda^{3\mu-6}$	$\Lambda^{4\mu-13}$	$\Lambda^{10\mu-13}$	$\Lambda^{20\mu-75/2}$
EPRL	$\Lambda^{3\mu-4}$	$\Lambda^{4\mu-9}$	$\Lambda^{10\mu-1}$	$\Lambda^{20\mu-15/2}$

Divergence as BF SU(2)

What about the Log?

Numerical confirmation

Easier than the asymptotic (fixed boundary)

Tensorial structure?

Crucial to setup a renormalization procedure (continuum limit)

Idea on how to compute it analytically

Conclusion and Outlook

Numerical evaluation of Spin Foam amplitudes is possible and a useful tool.

One vertex is possible

- Consistency check!
- Connection to the semi-classical limit

Two vertices is work in progress

- Bubbles are IR divergent
- Numerics can help us signing the path towards the continuum limit

Three vertices is in planning phase

- One full internal face is enough to have curvature (flatness problem)

Many vertices is a dream (realizable)

- Phenomenology?

Thanks for your attention!

Explicit form of Wigner dsmall

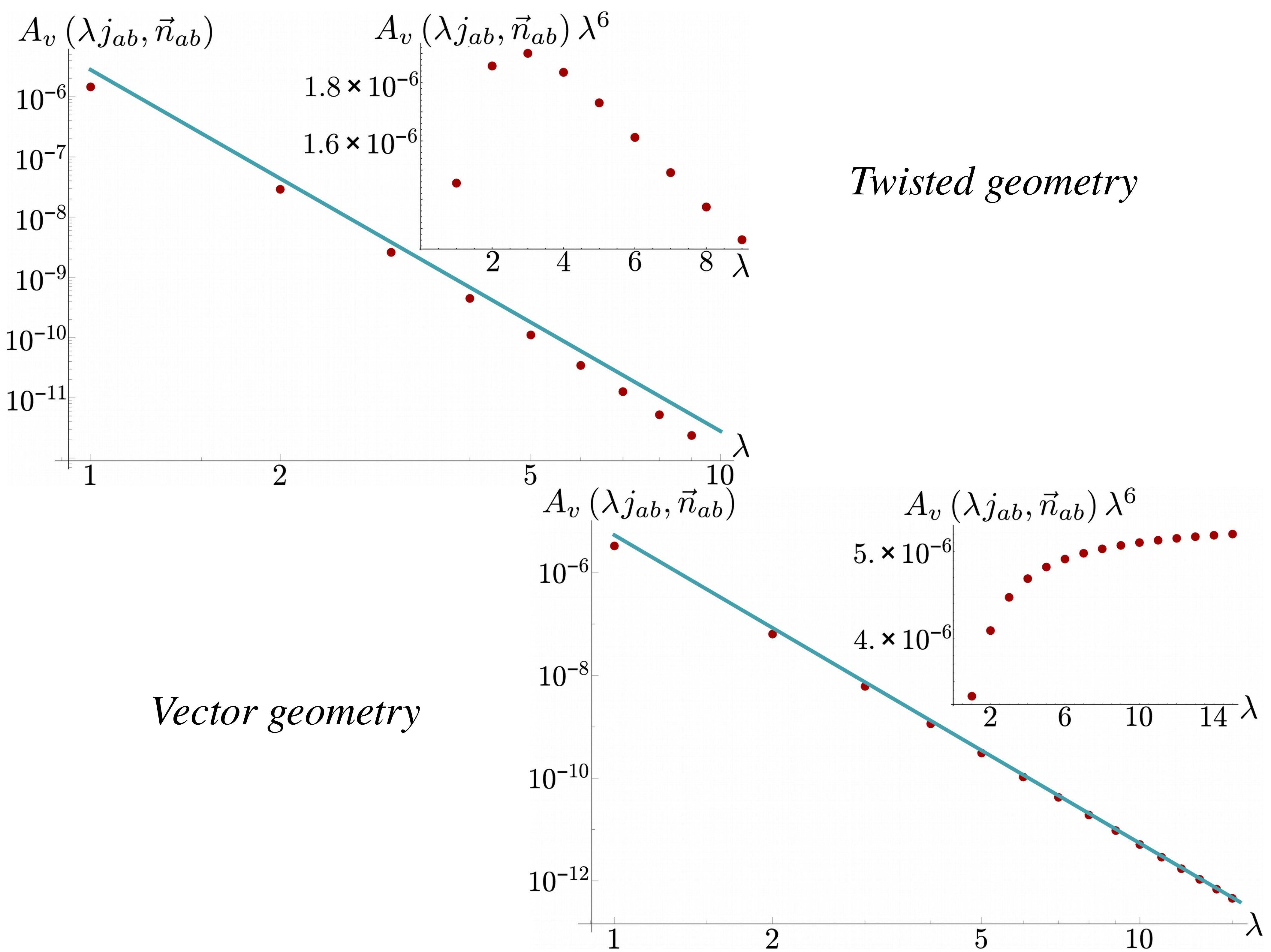
$$d^{(\gamma j,j)}_{jlp}(r) = (-1)^{\frac{j-l}{2}} \frac{\Gamma(j+i\gamma j+1)}{|\Gamma(j+i\gamma j+1)|} \frac{\Gamma(l-i\gamma j+1)}{|\Gamma(l-i\gamma j+1)|}$$

$$\frac{\sqrt{2j+1}\sqrt{2l+1}}{(j+l+1)!} \left[(2j)!(l+j)!(l-j)! \frac{(l+p)!(l-p)!}{(j+p)!(j-p)!}\right]^{1/2} e^{-(j-i\gamma j+p+1)r}$$

$$\sum_s \frac{(-1)^s e^{-2sr}}{s!(l-j-s)!} \, {}_2F_1[l+1-i\gamma j, j+p+1+s, j+l+2, 1-e^{-2r}]$$

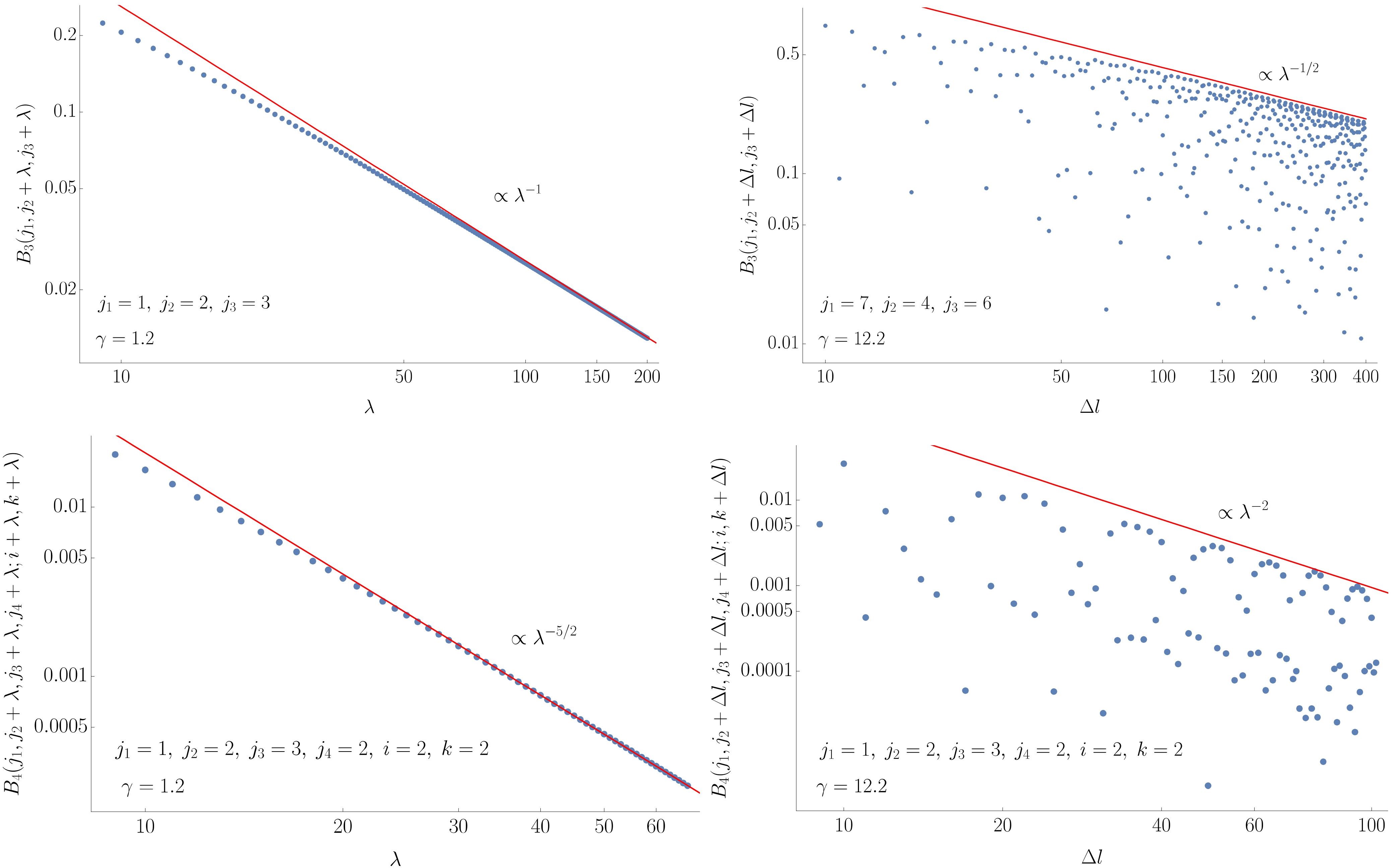
Generic SU(2) Invariant Asymptotic

Dofs	Geometry type	Saddle points	Behavior
5L-6N	twisted	0	Exponentially decreasing
3L-3N	vector *(anti-parallel)*	1	Power law decreasing without oscillations
	Conformal twisted *(angle-matching)*	2	Power law decreasing generalized Regge oscillations
2L-2N	Regge *(shape-matching)*	2	Power law decreasing generalized Regge oscillations
4N-10	polytope *(flat embedding)*	2	Power law decreasing Regge oscillations

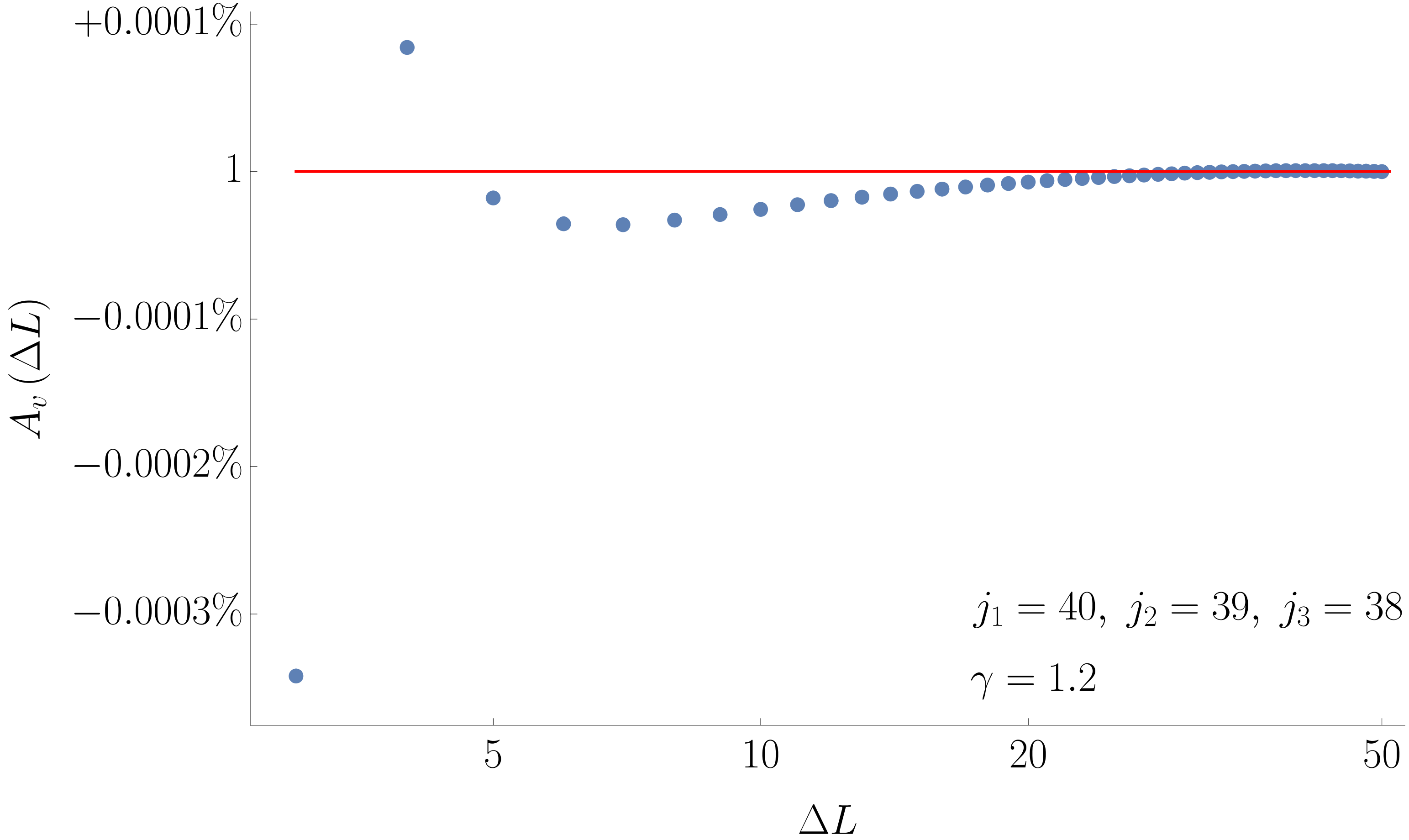


Twisted geometry
$A_v(\lambda j_{ab}, \vec{n}_{ab})$
$A_v(\lambda j_{ab}, \vec{n}_{ab})\lambda^6$
1.8×10^{-6}
1.6×10^{-6}
2 4 6 8 λ
10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 10^{-11}
1 2 5 10 λ
Vector geometry
$A_v(\lambda j_{ab}, \vec{n}_{ab})$
$A_v(\lambda j_{ab}, \vec{n}_{ab})\lambda^6$
$5.\times10^{-6}$
$4.\times10^{-6}$
2 6 10 14 λ
10^{-6} 10^{-8} 10^{-10} 10^{-12}
1 2 5 10 λ

Scaling of the Boosters

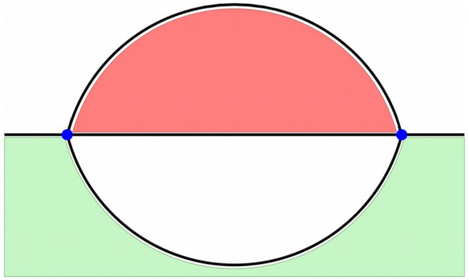


Convergence in the bubble



The 3D Bubble Explicitly

$$W^{\text{EPRL 3D}}_{\text{bubble}} = \sum_{j_1,j_2,j_3} \prod_{f=1}^{3} (2j_f+1)^{\mu} A_1 A_2,$$



Vertex Amplitude

$$A_v = \sum_{\Delta l_1,\Delta l_2,\Delta l_3} \begin{Bmatrix} k_1 & k_2 & k_3 \\ j_1+\Delta l_1 & j_2+\Delta l_2 & j_3+\Delta l_3 \end{Bmatrix} B_3(k_1, j_2+\Delta l_2, j_3+\Delta l_3)$$
$$B_3(j_1+\Delta l_1, k_2, j_3+\Delta l_3) B_3(j_1+\Delta l_1, j_2+\Delta l_2, k_3) \ .$$

Inferred from numerics:

$$\begin{Bmatrix} k_1 & k_2 & k_3 \\ \lambda_1 & \lambda_1 & \lambda_1 \end{Bmatrix} \propto \lambda_1^{-1/2}$$

$$B_3(k, \lambda+\Delta l, \lambda+\Delta l) \approx (\lambda)^{-\frac{1}{2}} (\lambda+\Delta l)^{-\frac{1}{2}}$$

Two bounded summations, one unbounded

$$A_v \approx \sum_{\delta_1} (\lambda_1+\delta_1)^{-\frac{1}{2}} (\lambda_1)^{-\frac{3}{2}} (\lambda_1+\delta_1)^{-\frac{3}{2}}$$

The amplitude scales as:

$$W^{\text{EPRL 3D}}_{\text{bubble}}(\Lambda) \approx \sum_{\lambda_1} \lambda_1^{3\mu} \left((\lambda_1)^{-\frac{3}{2}} (\lambda_1)^{-1} \right)^2 \approx \Lambda^{3\mu-4}$$