Quantum Gravity made from quantum polyhedra

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2017 Annual Meeting of the
APS Mid-Atlantic Section
5th November 2017

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Loop Quantum Gravity in One Slide

Problem addressed Describe the fundamental degrees of freedom when there is no fixed background space

How? Canonical quantization of the holonomy-flux algebra (SU(2) Gauge Theory)
(ADM splitting, Ashtekar connection and densitized triad phase space,
Dirac quantization of a constrained system)

Covariant quantization – Spin Foams
(Path Integral formulation, sum over geometries, canonical Hilbert space as boundary)

Who? Approximately 250 people and 50 groups (PI, PSU, CPT, Warsaw, ...)

What? Orthogonal Base (spin network states)
(graph Γ, spin j_l on each link, an intertwiner i_n on each node)

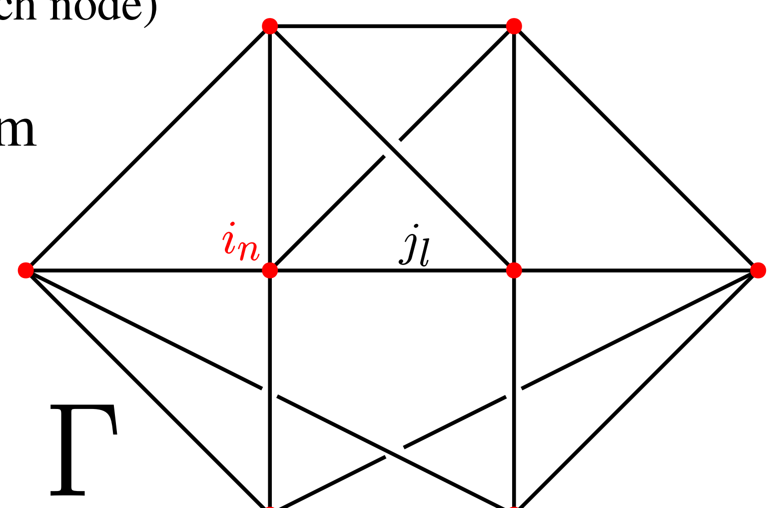
$$\mathcal{H} = \bigoplus_{\Gamma} \bigoplus_{j_l} \bigotimes_{n\in\Gamma} \mathcal{H}_{i_n} \quad \mathcal{H}_{i_n} = \mathrm{Inv}\left[\bigotimes_{l\in n} V_{j_l}\right]$$

Discreteness Geometric Operators have discrete spectrum

$$A(\Sigma) = \gamma\hbar G \sum_{l\in\Sigma} \sqrt{j_l\,(j_l+1)}$$

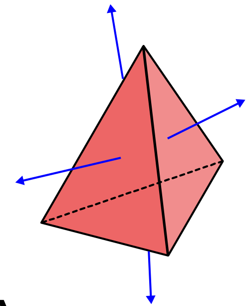
$$A_{\mathrm{gap}} = \frac{\sqrt{3}}{2}\gamma\ell_P^2$$

$$V(R) = (\gamma\hbar G)^{\frac{3}{2}} \sum_{i\in R} f\,(i_n, l_e)$$



Classical Polyhedron & Quantum Polyhedron

[Bianchi, P.D. and Speziale 2010]



Areas and face normals
are good variables

Minkowski Theorem $\{A_l\,, \vec{n}_l\}$ such that $\sum_{l=1}^{F} A_l\,\vec{n}_l = 0$

$\exists!$ convex polyhedron with these faces

Kapovich & Millson phase space

$$\mathcal{S}_F = \left\{ \vec{n}_l \in \left(S^2\right)^F \mid \sum_{l=1}^{F} A_l \vec{n}_l = 0 \right\} / SO(3)$$

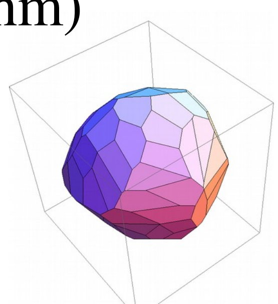
phase space of shapes of convex polyhedron with F faces

Asymptotic (soccer ball)

Reconstruction (Laserre algorithm)

Volume (Areas and normals)

Shape matching



Classical & Quantum Polyhedra

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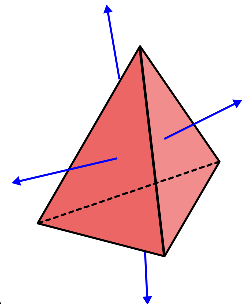

Areas and face normals are good variables

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Theorem $\exists !$ convex polyhedron with these faces

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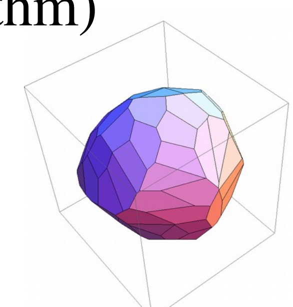


Guillemin-Sternberg Theorem

$$\begin{array}{ccc} A_l \vec{n}_l \quad \bigtimes_{l \in n} S^2_{A_l} & \longrightarrow & \bigotimes_{l \in n} V_{j_l} \qquad \vec{E}_l = 8\pi\gamma\ell_p^2 \vec{J}_l \\ \text{symplectic reduction} \Big\downarrow & & \Big\downarrow \text{quantum reduction} \\ \mathcal{S}_F & \longrightarrow & \mathcal{H}_{i_n} \end{array}$$

Quantum polyhedron state

[Livine and Speziale 2008]

$$||j_l, \vec{n}_l\rangle = \int_{SU(2)} \mathrm{d}g \bigotimes_{l \in n} g \triangleright |j_l, \vec{n}_l\rangle$$

Over-complete basis of $\mathcal{H}_{i_n}$

Good large spin limit (semi-classical)

Polyhedron properties as operators

Twisted Geometries

[Freidel and Speziale 2010]

Classical holonomy-flux configuration on a fixed graph describes a collection of polyhedra and their embedding

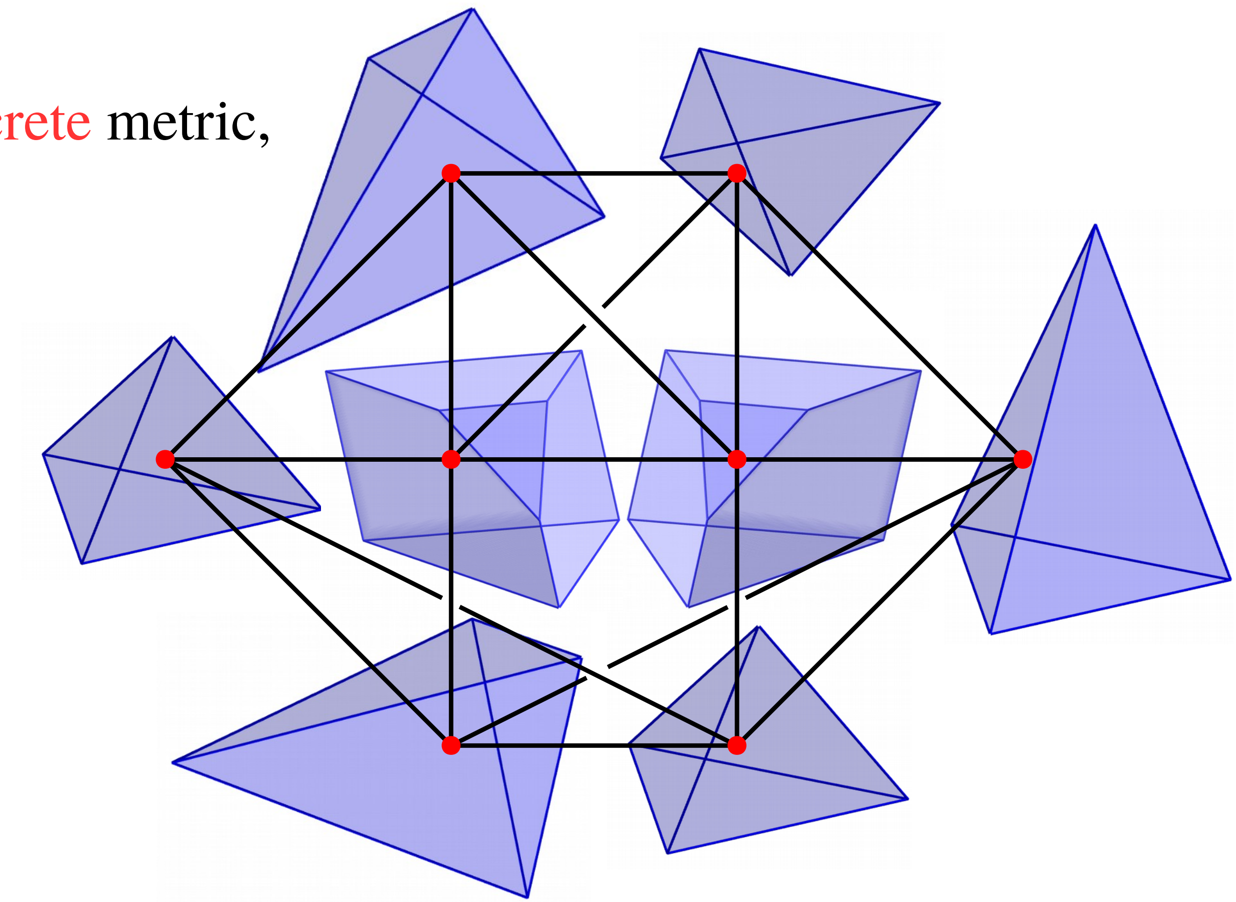
$$\mathcal{S}_\Gamma = \bigtimes_l \underset{j_l,\ \xi_l,}{T^*S^1} \bigtimes_n \underset{\text{shapes}_n}{\mathcal{S}_{F(n)}}$$

each polyhedron is locally flat: curvature emerges at the faces, as in Regge calculus

extrinsic geometry encoded in the parallel transport between adjacent polyhedra

the structure defines a piecewise discrete metric,
in general discontinuous
(same areas but different shapes)

Regge geometries are a sub-sector
(shape matched)



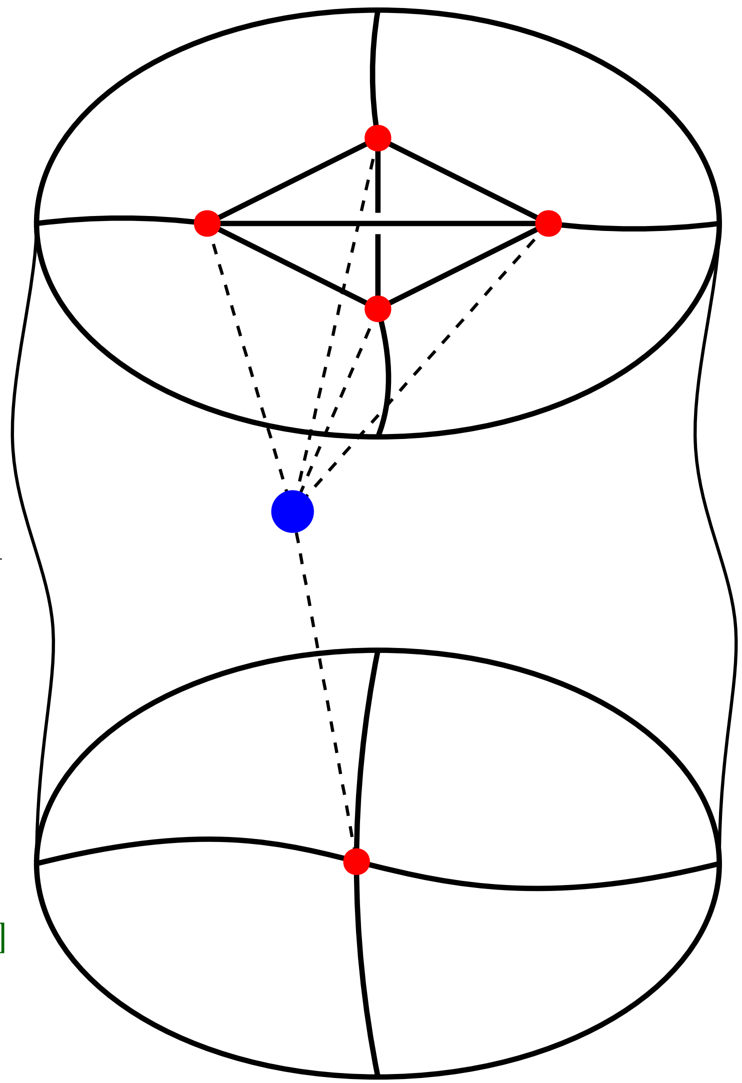
Dynamics with Spin Foams

Transition amplitude between spin networks

$$Z = \sum_{\sigma} \sum_{j_f} \prod_{f} (2j_f + 1) \prod_{v} A_v^{EPRL}(j_f, i_e)$$

2-complex decorated with SL(2,C) irreps

(defined in 3+1 dimensions, Lorentzian signature)



Can be derived in a number of different ways, based on a definition of GR as a constrained topological theory

(principal series irreps)

The model currently used is the EPRL model

[Engle, Pereira, Rovelli and Livine, 2008]

Key result

$$A_v^{EPRL}(j_f, i_e) \quad \xrightarrow{j\to\infty} \quad \exp\left(i S_{\text{Regge}}\right)$$

sum over histories of spin networks weighted by exponentials of discretized GR

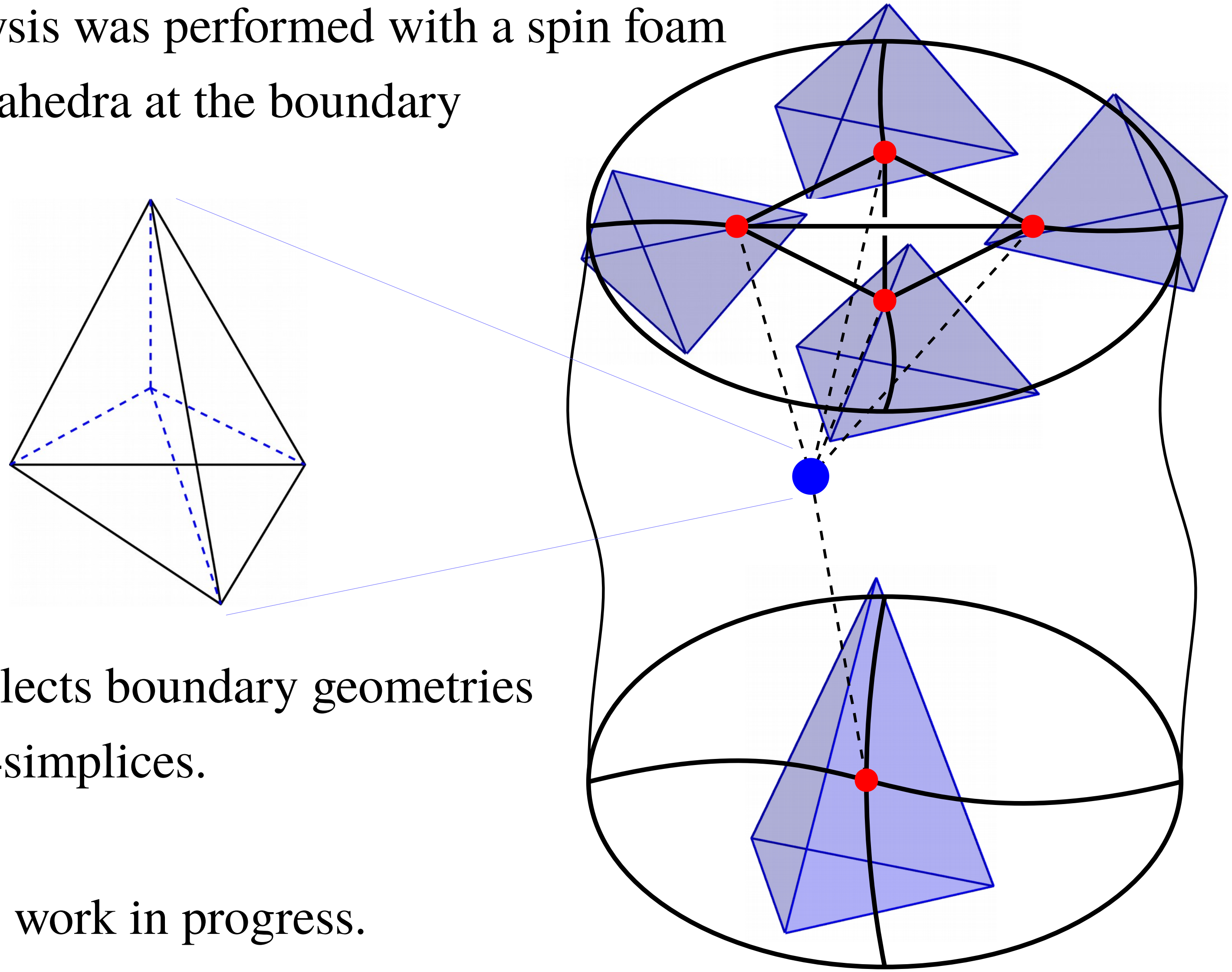
[Barrett & collab. 2010]

[PD, Fanizza, Sarno and Speziale 2017]

[Bahr and collab. 2015]

4D Polytopes

Asymptotic analysis was performed with a spin foam vertex with 5 tetrahedra at the boundary



The amplitude selects boundary geometries that reconstruct 4simplices.

Generalization is work in progress.

We have indication for richer geometries

[PD, Fanizza, Sarno and Speziale 2017]

Conclusion and Outlook

The geometrical interpretation of Canonical LQG Hilbert space on a fixed graph is fulled understood in terms of a collection of flat polyhedra (discontinuous metric) the curvature is concentrated on faces. Twisted Geometries picture.

The dynamic of the theory is GR in the semi-classical limit (Regge calculus), many open questions (generalization, extended tessellations, continuum limit).