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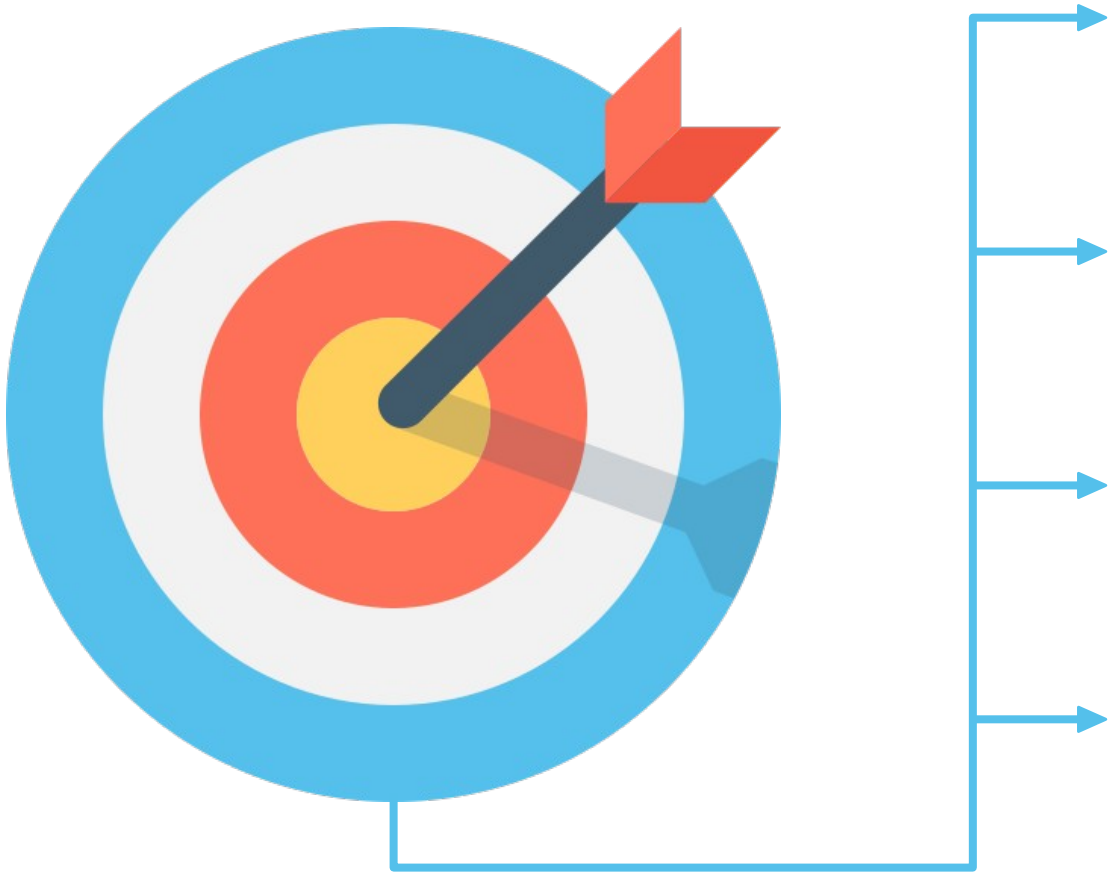
Spin foam theory.
What is it?
Where are we?

Group seminar Ludwig-Maximilians-Universität, Munich - 19th May 2023

Goals



What is a spin foam?

What do we know?

What do we not know?

What am I working on?

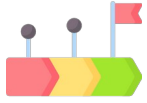
I am strongly biased

What is a spin foam?

[C. Rovelli and F. Vidotto – 2014]

(Tentative) path integral quantization of General Relativity

Background independent
& Lorentz covariant



Dynamics for LQG
(transition amplitude)

Strong analogy with 1D
quantum mechanics

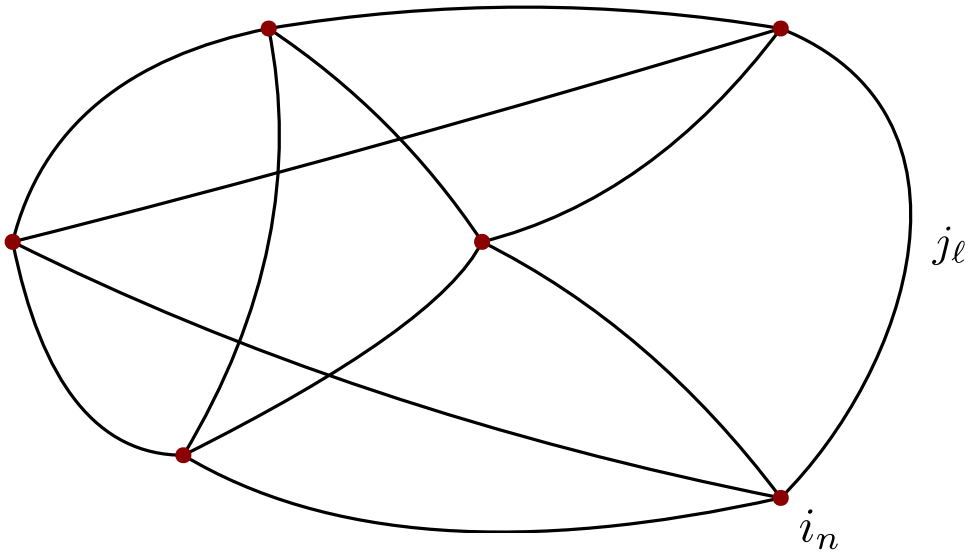
Region of space-time

Boundary space-like surface

Quantized a la LQG

Boundary space

Gravitational degrees of freedom (holonomies and fluxes) truncated on an abstract graph



Kinematical Hilbert space

$$\mathcal{H}_\Gamma = L^2(SU(2)^L/SU(2)^N)$$

Geometric quantities becomes operators

- e.g. areas, volumes, angles
- discrete spectrum

Spin networks form a basis (CSCO)

- graph, spins and intertwiners

$$|\Gamma,\ h_\ell\rangle = \psi(h_\ell) \quad \longleftrightarrow \quad |\Gamma,\ j_\ell\ i_n\rangle$$

Boundary space

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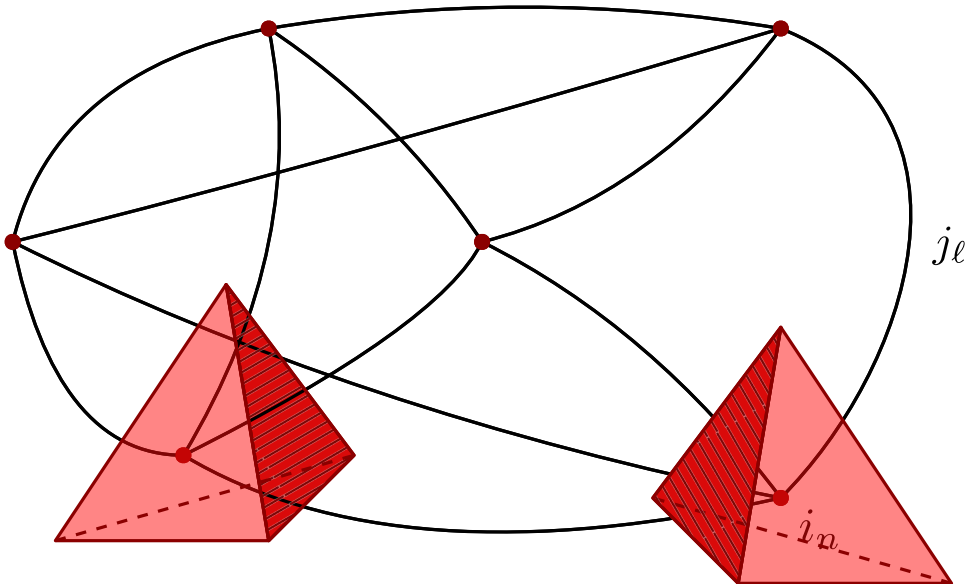

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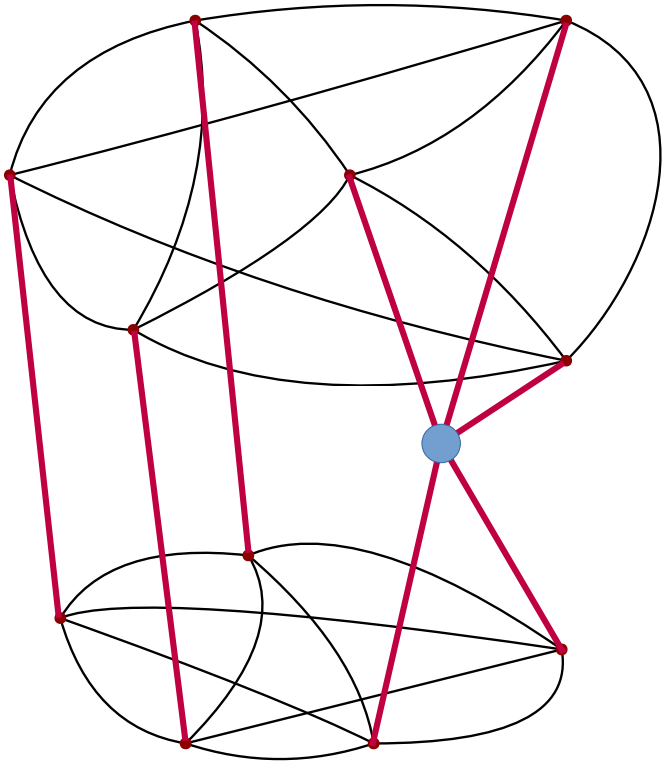
Geometric interpretation: discrete chunks of quantum space

Region of space-time



Example of a 2-complex between two simple spinnetworks with only one vertex

Truncate the space-time degrees on freedom with a 2 complex (vertices, edges, faces) compatible with the boundary graph.

5 valent vertex as elementary building block

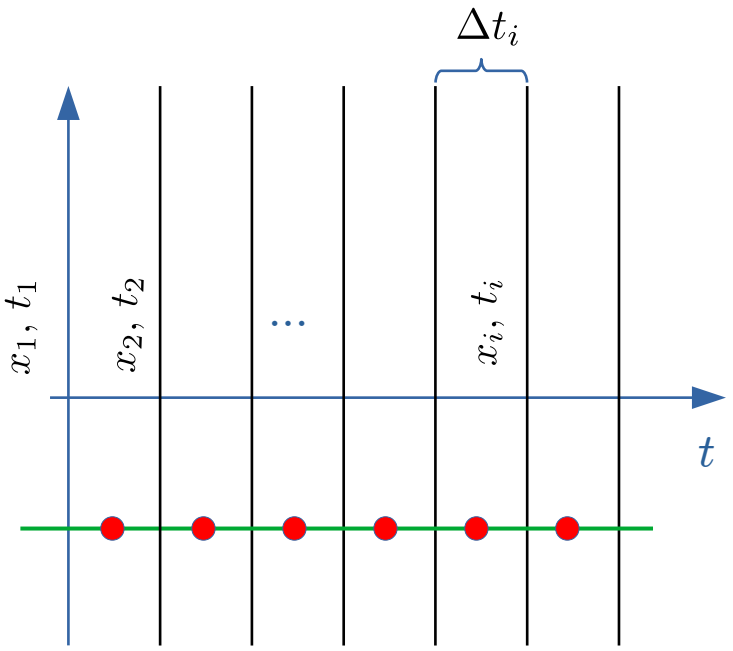
2-complex colored by LQG quantum numbers

Assign local amplitudes (spin network basis)

$$Z_\Delta = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v\,(j_f,\ i_e)$$

Quantum Mechanics

Textbook story, exotic language...



Regularize by discretizing time, 1D time lattice, consider the dual (vertices and edges)

Edge Hilbert space (initial and final state – position eigenstate)

$$\psi(x) = \delta(x - x_i)$$

Local amplitudes (propagators) to each vertex (e.g. free particle)

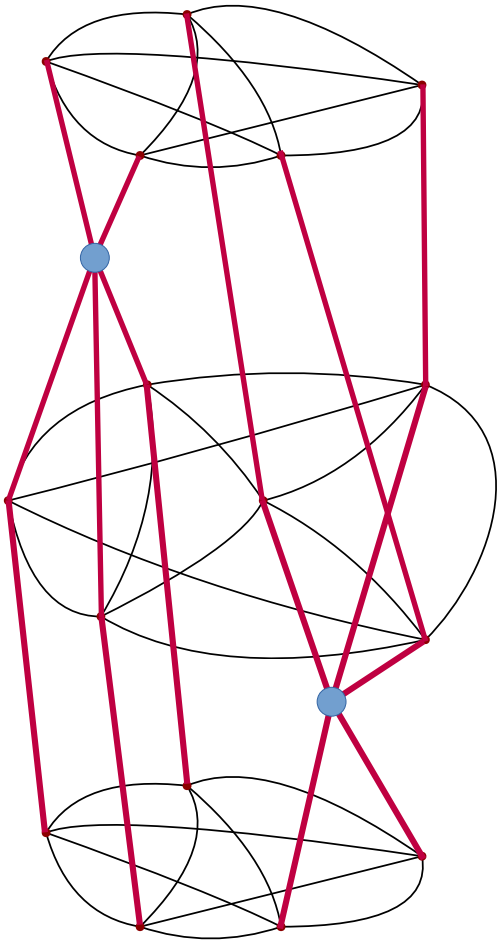
$$K(x_i, x_{i+1}; \Delta t_i) = \sqrt{\frac{m}{2\pi i \Delta t}} \exp\left[\frac{im}{2\Delta t}(x_i - x_{i+1})^2\right]$$

Transition amplitude between two position in one time step

$$x_i \longleftrightarrow x_{i+1}$$

Partition function is the product of all vertex amplitudes integrate on all intermediates states

Region of space-time



Vertex amplitude is the elementary propagator

$$Z_\Delta = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v\,(j_f,\ i_e)$$

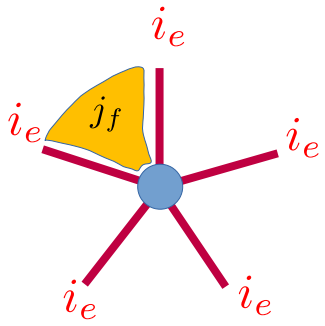
Compose propagators and sum over the quantum numbers of intermediate states

Transition amplitudes between LQG states, sum over histories!

The EPRL model [J. Engle, R. Pereira, C. Rovelli and E. Livine – 2009]

$$A_e(i_e) = 2i_e + 1$$
$$A_f(j_f) = 2j_f + 1$$

$$A_v\left(j_f,\ i_e\right) = \int \prod_{e\in v} \mathrm{d}g_e\, \delta(g_1) \sum_m \prod_{f\in v} D^{\gamma j_f, j_f}_{j_f m_{f_t}, j_f m_{f_s}}(g_t^{-1} g_s) \prod_{e\ni f} \begin{pmatrix} j_f \\ m_p \end{pmatrix}^{(i_e)}$$



One Lorentz holonomy to each face (parallel transport)

Derived from BF topological theory + simplicity constraints

Unitary Irreducible representation (weak implementation of simplicity constraints)

Require trivial parallel transport within the vertex (flat building blocks)

Integrate over all the possible parallel transports (regularized)

What do we know?

Semiclassical limit of the EPRL theory

Large quantum numbers regime

$$A = 8\pi\gamma\,\hbar G\,j$$

$$\hbar \ll 1 \quad \longleftrightarrow \quad j \gg 1$$

$$\hbar G = 1$$

Path integral dominated by classical trajectories

Vertex amplitude $A_v \approx e^{iS_R}$ Lorentzian 4-simplices

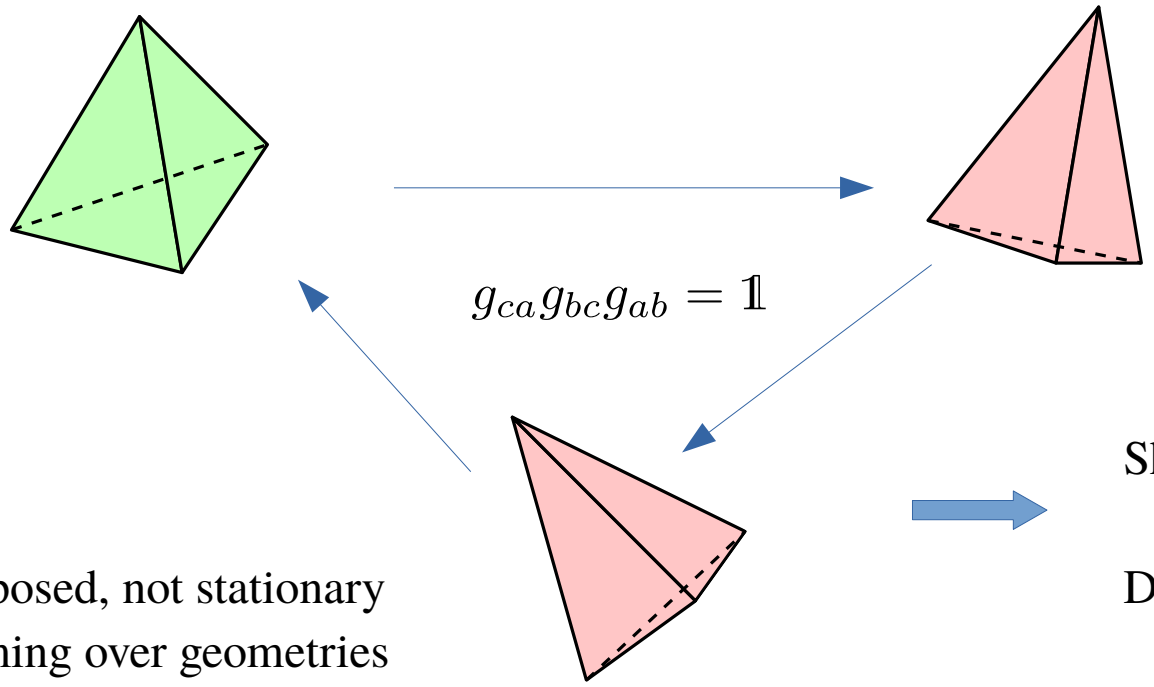
[J. Barrett, ... – 2011]

Partition function?

Local flatness

[P.D. – 2022]

Trivial parallel transport within the vertex (flat building blocks)



Strongly imposed, not stationary phase, summing over geometries as holonomies

Shape matching
+
Diehedral angles
=
Lorentzian spacelike simplices

Deconstruction

[P.D. – 2022]

Local flatness

Emergence of local Lorentzian geometry

Model independent

strongly imposed

Simplicity constraints

Gluing into global geometries

=

Regge-like holonomies

Perfect gluing

$$g_f = e^{\Theta \vec{n} \cdot \frac{\vec{\sigma}}{2}}$$

Alignment property

saddle point

Partition function

Extract the angles to form the Regge action

Deconstruction

[P.D. – 2022]

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Alignment property

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Partition function

Extract the angles to form the Regge action

Constrain the face holonomies

$$g_f = e^{(\Theta + i\xi)\vec{n} \cdot \frac{\vec{\sigma}}{2}}$$

$$\gamma\Theta + \xi = 0 \ (mod 4\pi)$$

flatness problem?

Distribution analysis

EPRL specifc

Possible paths

Local flatness

Emergence of local Lorentzian geometry

Simplicity constraints

Gluing into global geometries

Partition function

Constrain the face holonomies

Change the definition of the semiclassical limit into a double limit (large spins and small/refined curvature)

$$\gamma j \Theta \ll 1$$

What am I working on?

Numerical calculations of spin foams

Why numerics?

More tools the better

Explorative

How numerics?

sl2cfoam-next

Who numerics?

Myself, Pietropaolo Frisoni, Jared Wogan

Where numerics?

sl2cfoam-next

Open source library to compute EPRL amplitudes

[F. Gozzini – 2021]
[P.D and G. Sarno – 2018]

Modular (divide & conquer)

Optimized for HPC

User friendly (Julia scripting interface)

Resource demanding (relatively low spins – hybrid technique)

[M. Han ... – 2021]
[S. Steinhaus ... – 2022]

Unavoidable approximation (technical downside)

Applications

A step by step tutorial (summary)

- How-To compute a spin foam amplitude?

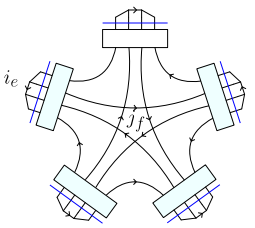
[P.D. Pietropaolo Frisoni 2022]

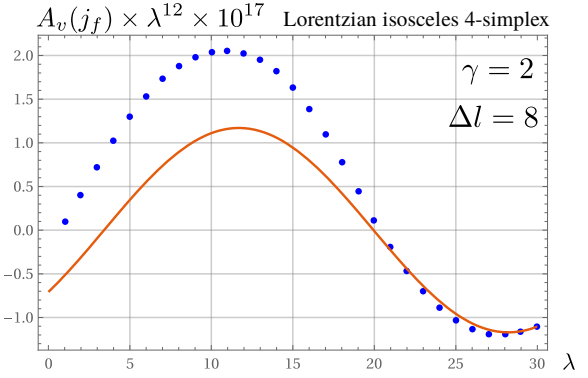
```
using SL2Cfoam
Immirzi = 1.2
data_folder = "path_data_folder"
configuration = SL2Cfoam.Config(VerbosityOff, VeryHighAccuracy, 100, 0)
SL2Cfoam.cinit(data_folder, Immirzi, configuration)
boundary_spins = ones(10)
Dl = 10
Av = vertex_compute(boundary_spins, Dl)
```

Verified single vertex asymptotics (confirmation)

- Oscillations governed by the Regge action

[P.D. Marco Fanizza, Giorgio Sarno, Simone Speziale 2019]

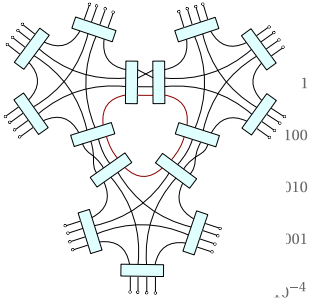


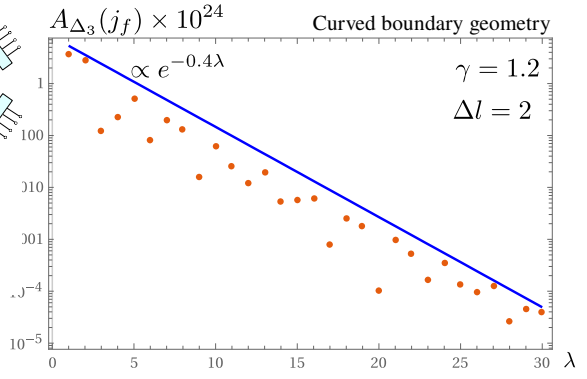


Three vertices and the “flatness problem” (decisive)

- Amplitudes of curved geometries are suppressed
- Semiclassical limit: large spins and small curvature

[P.D. Francesco Gozzini, Giorgio Sarno 2020]





Bubble divergences (new questions)

- Numerical estimates of many IR divergences
- Linear divergence (2-vertices, 6 faces), Convergence (5-vertices, 10 faces), Why?

[P.D. Pietropaolo Frisoni 2023]
[P.D. Pietropaolo Frisoni, Edward Wilson-Ewing 2022]

What do we not know?

Semiclassical limit of the EPRL theory

Double limit?

Einstein equations?

Refinement?

Predictions?

Matter?

What do we not know?

Semiclassical limit of the EPRL theory

Double limit? → Numerical quantitative exploration

Einstein equations? → Define a new spin foam model

Predictions? → Tunneling of quantum geometries?

Summary

Spin foam theory provides dynamic to LQG states
(path integral as in QM, assign transition amplitude)

Connection with (discrete) GR in the $\hbar \ll 1$ limit

- Single vertex (propagator) works
- The sums over bulk dof has some problems
  (we had to revise the semiclassical limit as a double limit)

Numerics could be an important new tool

- “Experiments”

Many open questions! The work is not done!

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Questions?