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An (over)view of covariant loop quantum gravity

Gravity, Astroparticle, and Particle Physics (GAPP) seminar
PennState - 17th February 2023

(Over)view of covariant loop quantum gravity

What, Why, When, Where, Who of this talk

Path integral for General Relativity in LQG variables

(Over)view = Focus on the logic less on details. What do we do?

My view = Not a derivation, my way of see things I digested in years (not 100% shared), a lot of body language.

I'm happy to answer focused and technical questions if necessary

(Over)view of covariant loop quantum gravity

What, **Why**, When, Where, Who of this talk

Overcome difficulties in the LQG quantization program (dynamics)

Connection with second quantization techniques (Group Field Theories)

Very appealing geometrical interpretation and connection with Regge Calculus

(Over)view of covariant loop quantum gravity

What, Why, **When, Where, Who** of this talk

The theory is 20-25 years old, recent interest because of numerical calculations

Many people contributed

POC: Carlo Rovelli (UWO, CPT Marseille, PI) – Phenomenology

Muxin Han (FAU) – Semiclassical limit

Myself (UWO, Marseille) – Numerical calculations

Bianca Dittrich (Perimeter institute) – Classical theory

… if I have to put labels

(Over)view of covariant loop quantum gravity

We have an interesting path integral quantization of GR.

Many features, many open questions.

Covariant loop quantum gravity

Background independent Lorentzian path integral quantization of General Relativity

Partition function:

$$Z_\Delta = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v\left(j_f,\ i_e\right)$$

Covariant loop quantum gravity

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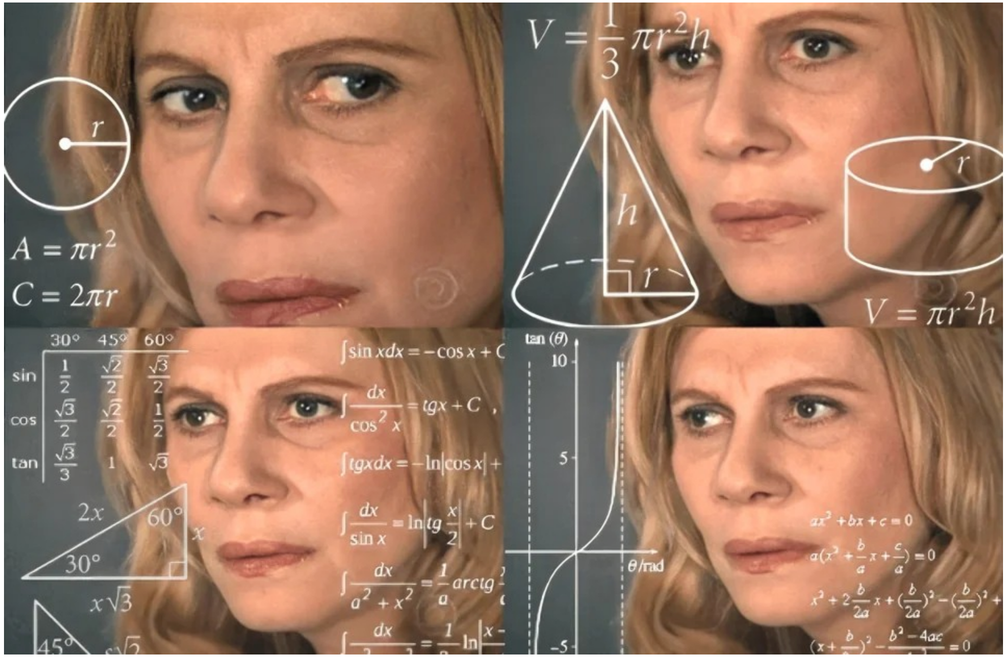
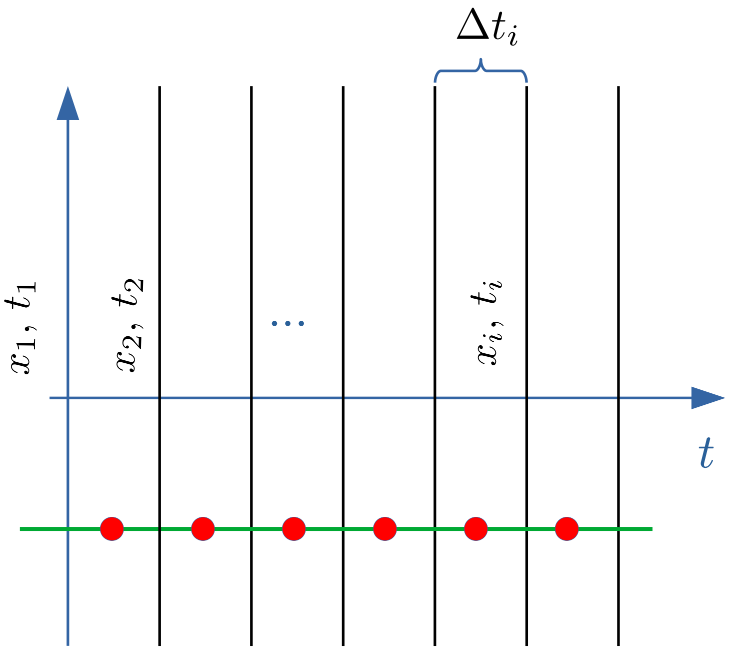
Partition function: $$Z_\Delta = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v\,(j_f,\ i_e)$$

How?



Quantum Mechanics

Textbook story, exotic language...



Regularize by discretizing time, 1D time lattice, consider the dual (vertices and edges)

Edge Hilbert space (initial and final state – position eigenstate)

$$\psi(x) = \delta(x - x_i)$$

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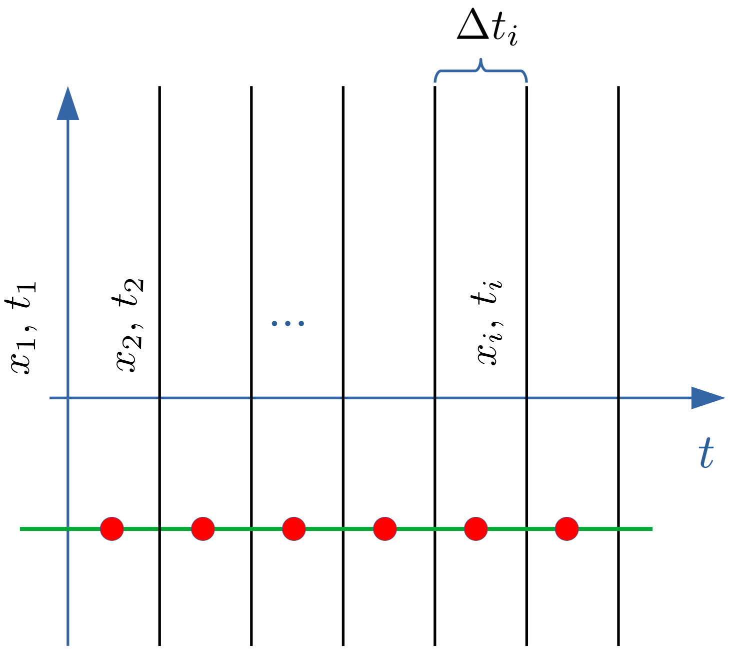

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Local amplitudes (propagators) to each vertex (e.g. free particle)

$$K(x_i, x_{i+1}; \Delta t_i) = \sqrt{\frac{m}{2\pi i \Delta t}} \exp\left[\frac{im}{2\Delta t}(x_i - x_{i+1})^2\right]$$

Transition amplitude

$$x_i \longleftrightarrow x_{i+1}$$

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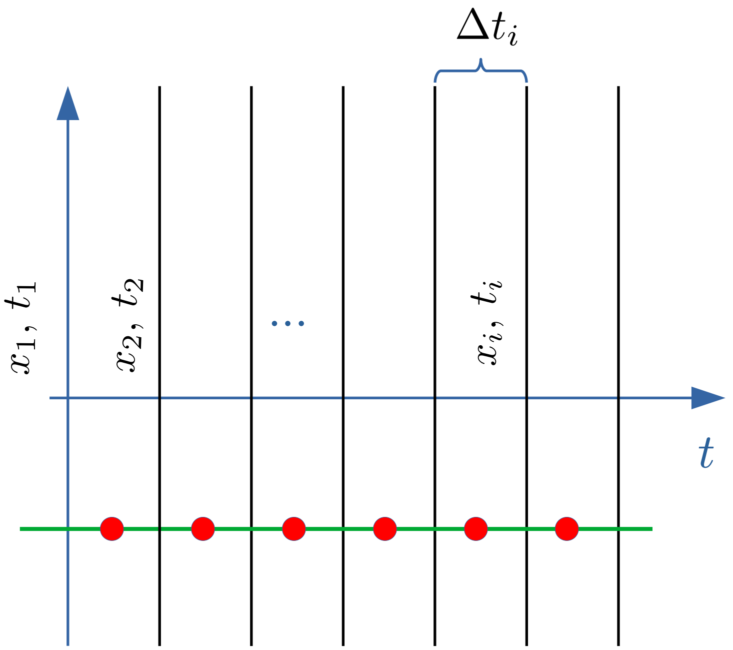

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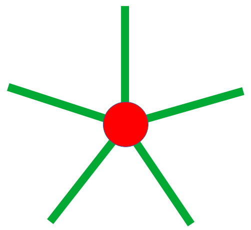
Partition function is the product of all vertex amplitudes integrate on all intermediates states

Regularize the path integral

We discretize the spacetime manifold with the 2-complex made with (dual) Lorentzian 4-simplices with spacelike boundary (5 tetrahedra) (d.o.f. truncation)



Fundamental building block (vertex with 5 edges)

Glue them together connecting edges

Combinatorial information (no geometric data)

In quantum mechanics (vertex with 2 edges)

Regularize the path integral

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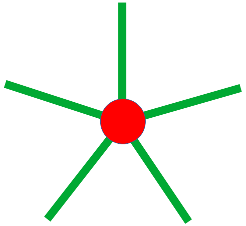

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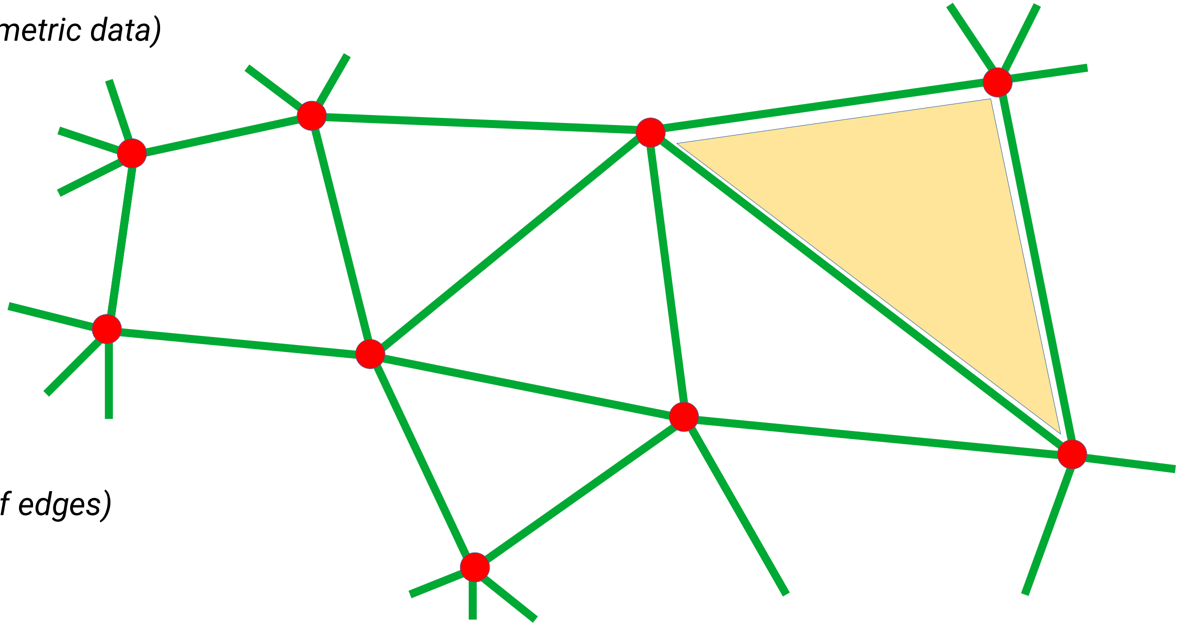
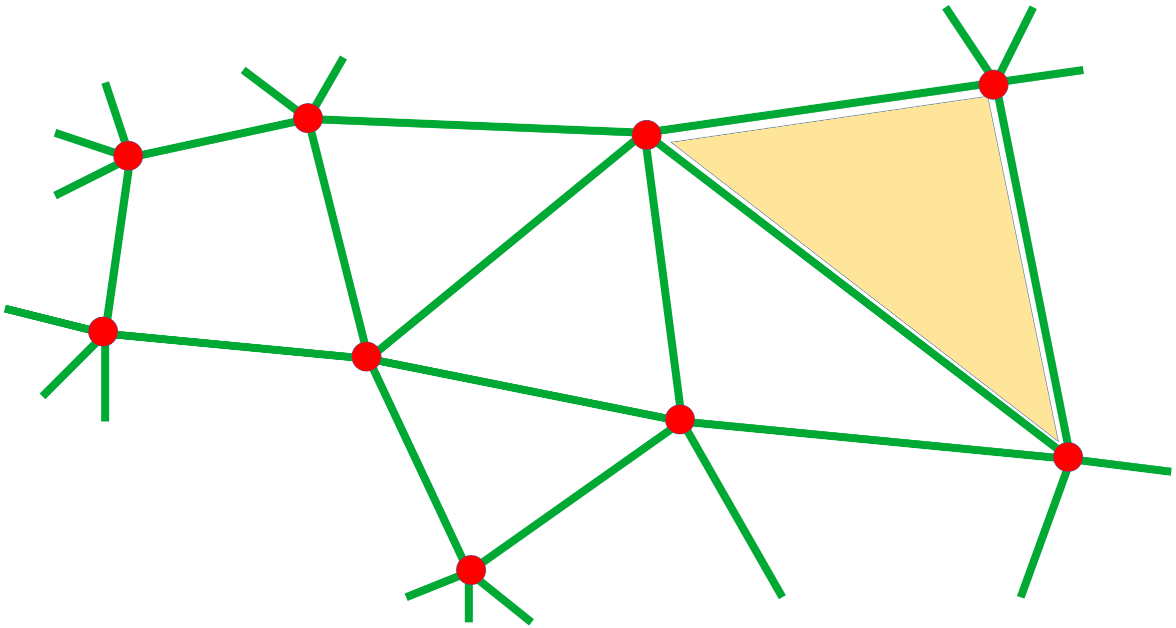


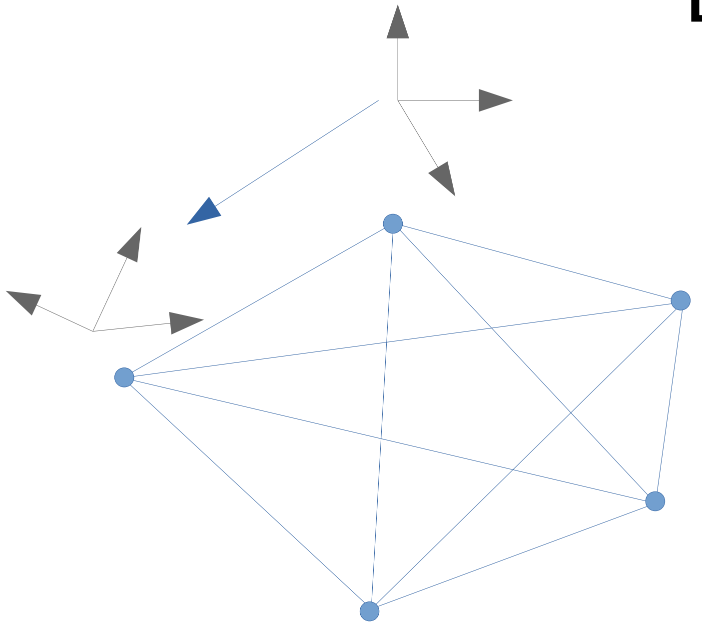
We form faces (closed collection of edges)

Regularize the path integral

$$Z_{\Delta} = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v\,(j_f,\ i_e)$$



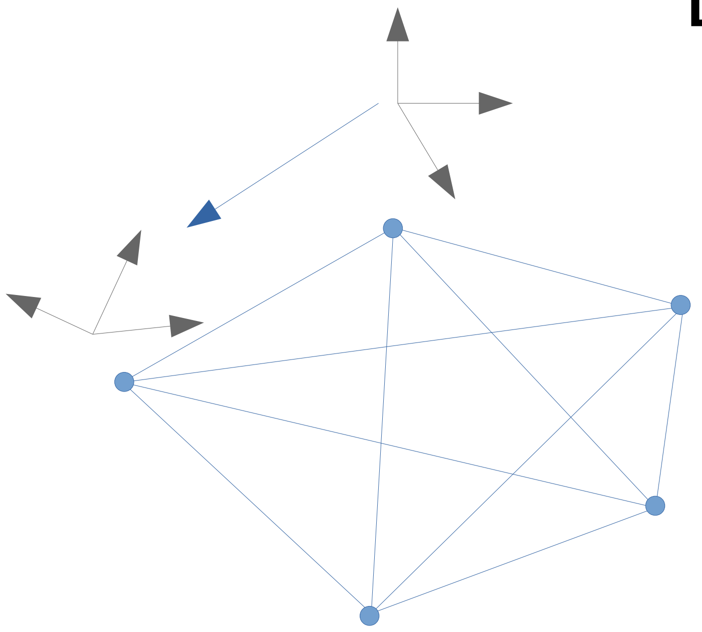
LQG Hilbert Space



Quanta of the gravitational field describing the geometry of a spacelike surface (truncation = graph). Canonical quantization.

Tetrad formulation (local reference frames) and parallel transports given by the SU(2) Ashtekar holonomy

LQG Hilbert Space



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Geometric quantities (areas, volumes, angles) are operators with discrete spectrum

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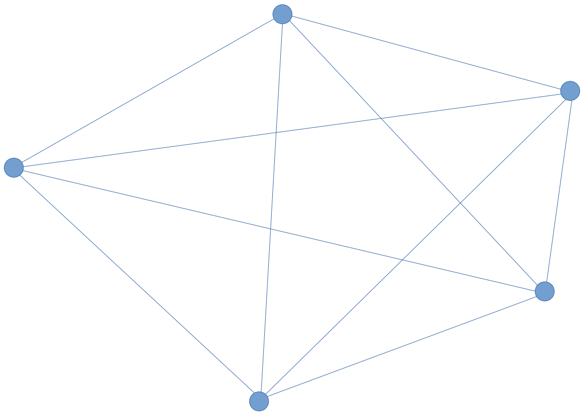

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Spinnetwork basis (spins and intertwiners)

j_f *Spins color links of the graph. Links carry quanta of area*

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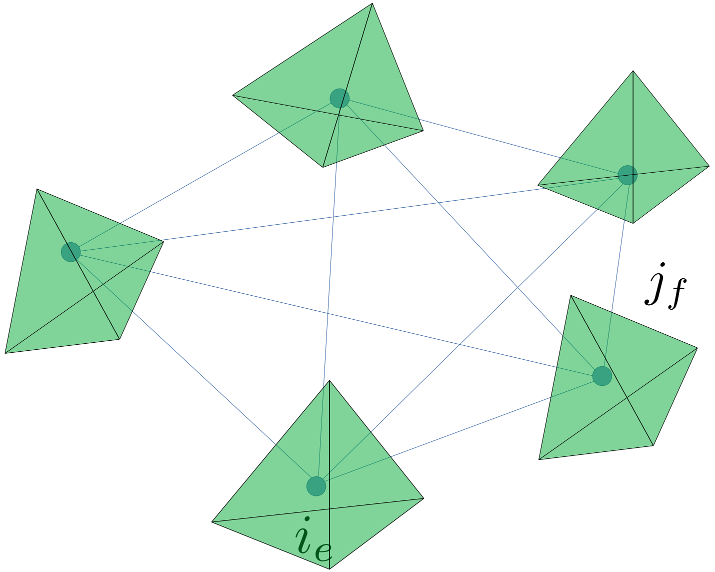

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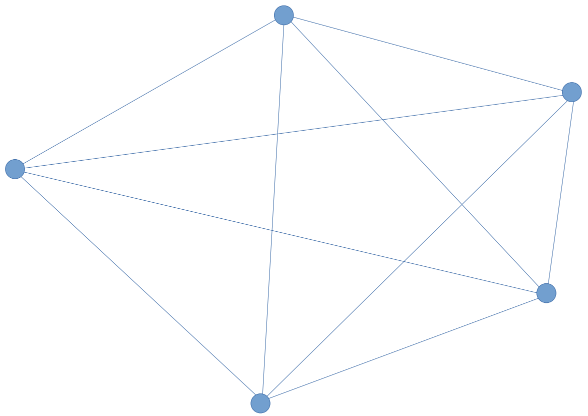
Geometric picture – grain of space with quantized volume and areas (fuzzy quantum polyhedra)

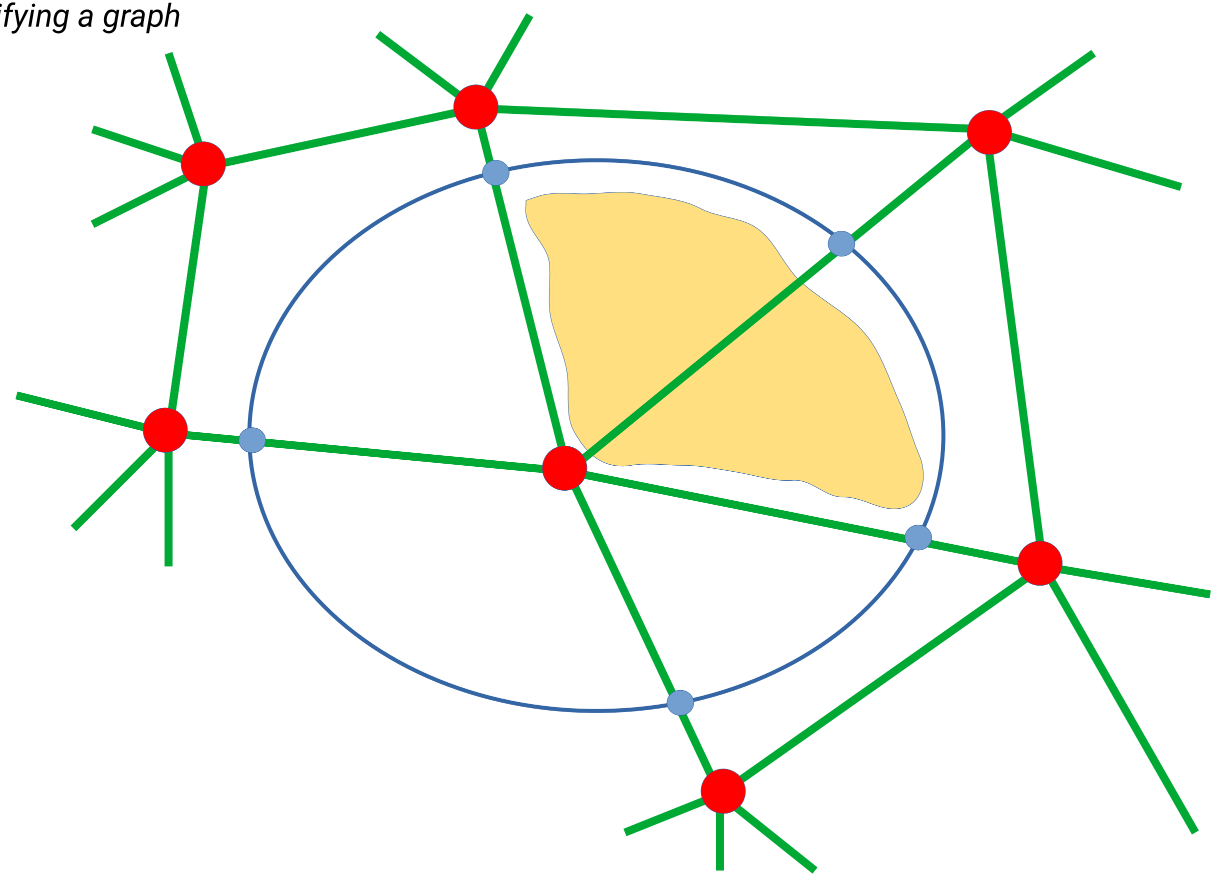
[E. Bianchi, P.D and S. Speziale – 2010]

Hilbert Space in the LQG Path Integral

Any spacelike surface intersect a number of edges, identifying a graph

The graph is dual to a triangulation of a spacelike surface





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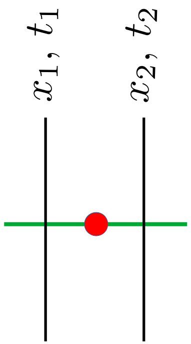
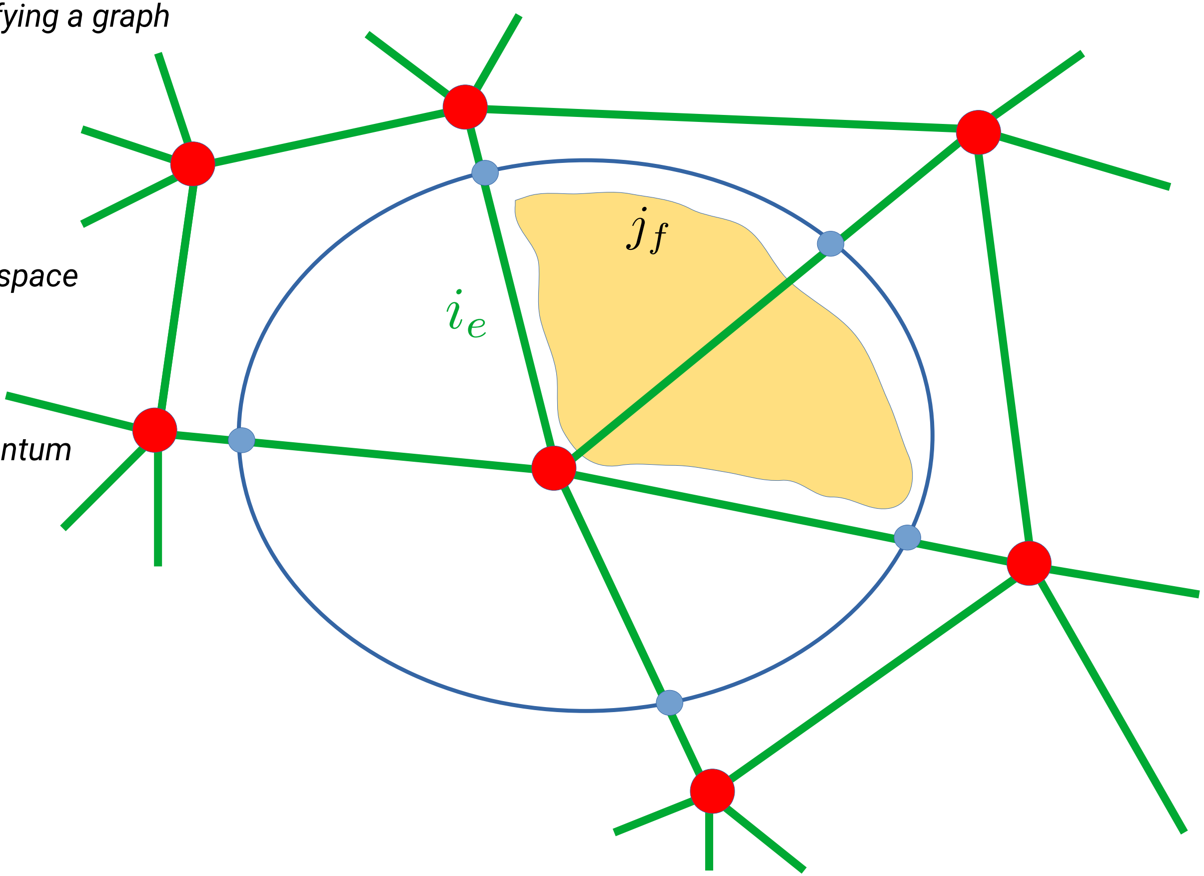
Associate to the boundary of every vertex the LQG Hilbert space (quantum states of the spacelike surface)

The Spinnetwork basis colors the 2-complex with LQG quantum numbers





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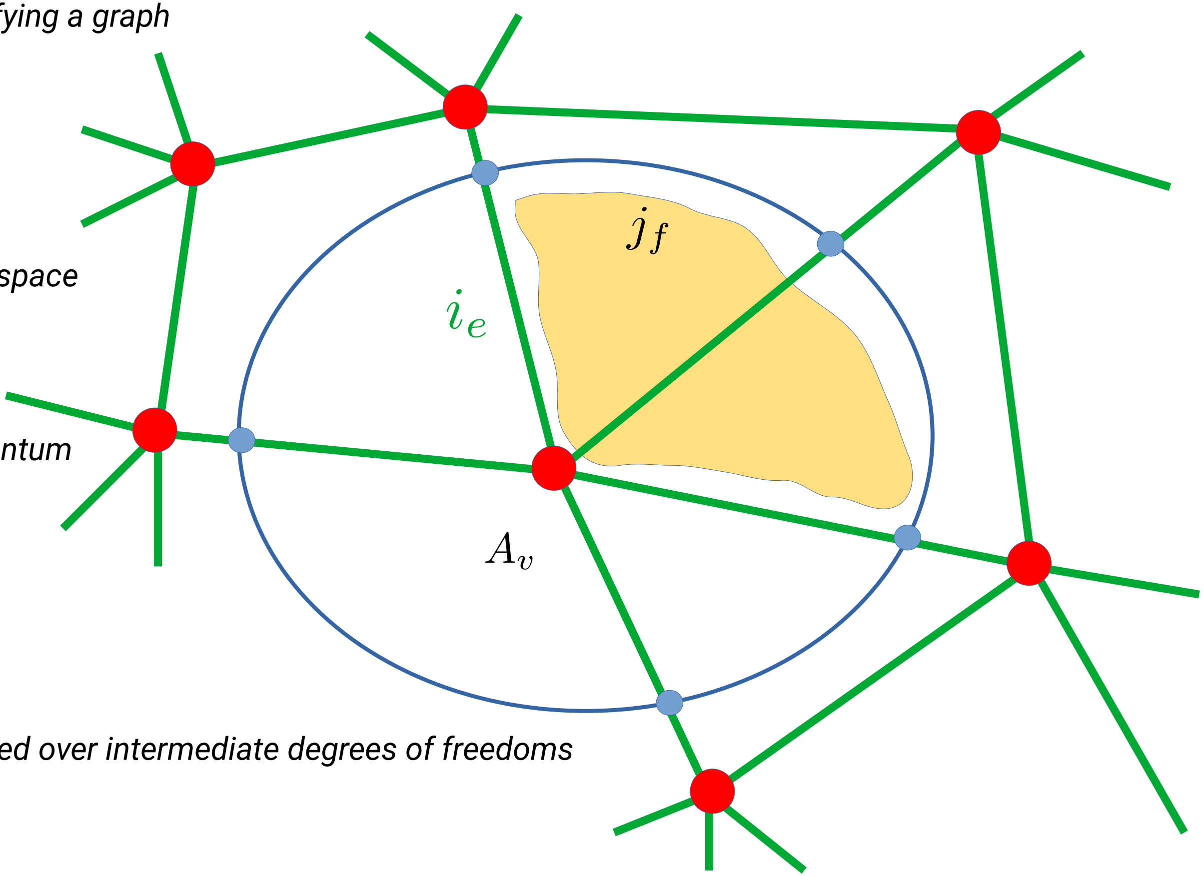
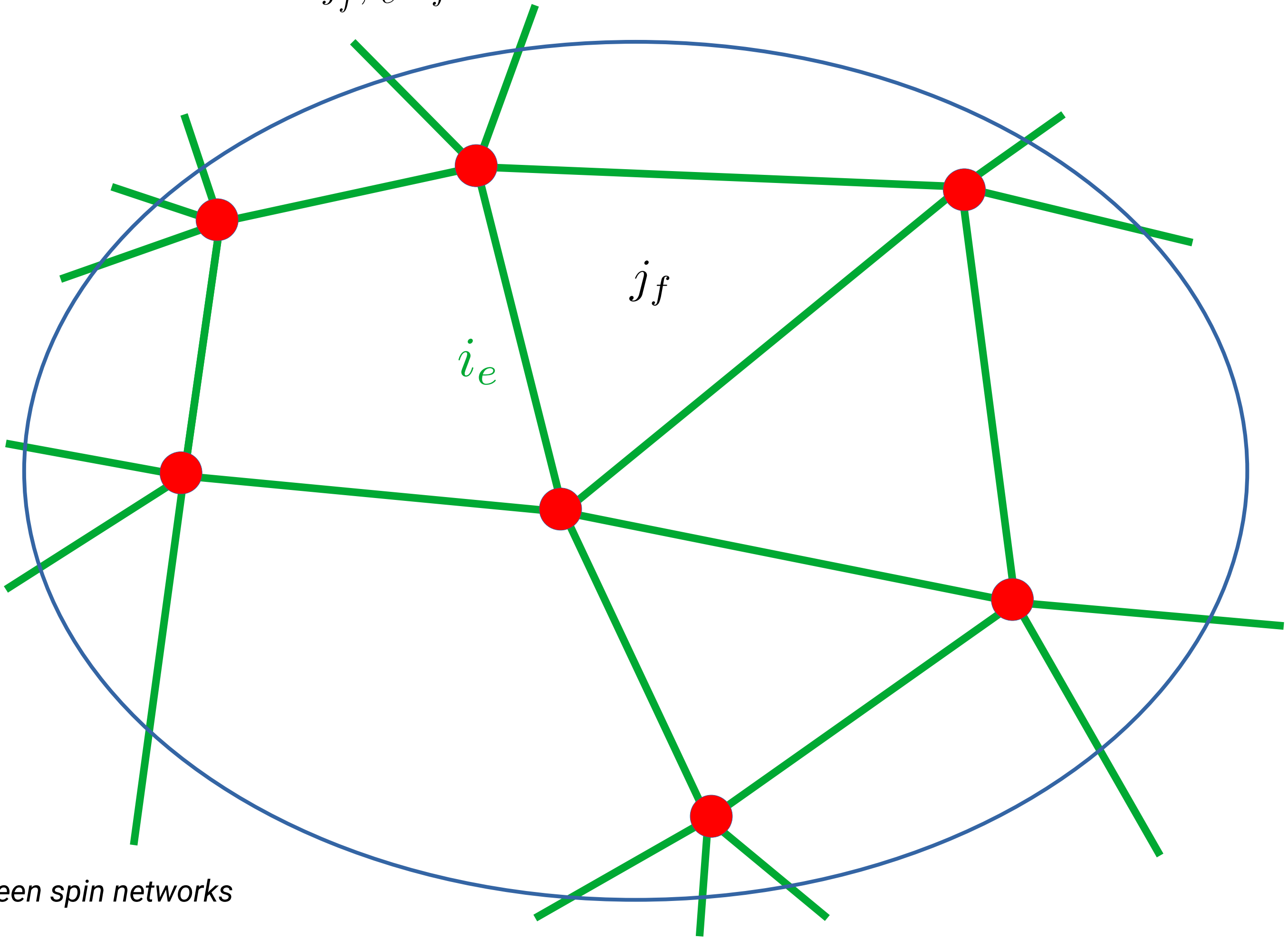
Attach an amplitude to each vertex.

Partition function as the product of local quantities summed over intermediate degrees of freedoms (sum over histories of spacelike surfaces)



TLDL: $$Z_{\Delta} = \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(i_e) \prod_v A_v(j_f,\ i_e)$$



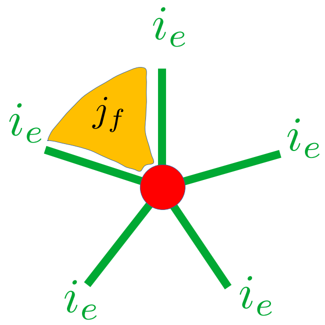
Transition amplitude between spin networks

The EPRL model

[J. Engle, R. Pereira, C. Rovelli and E. Livine – 2009]

$$A_e(i_e) = 2i_e + 1$$
$$A_f(j_f) = 2j_f + 1$$

$$A_v\left(j_f,\ i_e\right) = \int \prod_{e\in v} \mathrm{d}g_e\, \delta(g_1) \sum_m \prod_{f\in v} D^{\gamma j_f, j_f}_{j_f m_{f_t}, j_f m_{f_s}}(g_t^{-1} g_s) \prod_{e\ni f} \begin{pmatrix} j_f \\ m_p \end{pmatrix}^{(i_e)}$$



Boundary spin network with five nodes

One Lorentz holonomy to each face (parallel transport)

Specific choice of Unitary Irreducible representation (weak implementation of simplicity constraints $\vec{K} = \gamma \vec{L}$*)*

Require trivial parallel transport within the vertex (flat building blocks)

Integrate over all the possible parallel transports (regularized)

Is it so complex! Can you compute it?

Formal calculations, analytic approximations in certain regimes, numerical codes are available.

sl2cfoam-next

[F. Gozzini – 2021]

[P.D and G. Sarno – 2018]

Open source library to compute EPRL amplitudes

Modular (divide & conquer)

Optimized for HPC

User friendly (Julia scripting interface)

Monte Carlo techniques to sum over quantum numbers (two weeks ago)

[P.D and P. Frisoni – 2023]

Resource demanding (relatively low spins are accessible – different technique)

[M. Han ... – 2021]

Unavoidable approximation (technical downside)

A lot more to learn and improve

Can we recover the GR partition function?

In QM solutions of classical equations of motions dominate the path integral $\hbar \to 0$

Area spectrum

Stationary phase analysis in the limit of homogeneous rescaling of the boundary spins $\hbar G \sqrt{j_f(j_f+1)}$

Expectations discrete GR (semiclassical limit not the continuum limit) = 4D Regge calculus $S_R = \sum_t A_t(\ell_s)\theta_t(\ell_s)$

No matter! Pure gravity!

No cosmological constant!

Connection with discrete GR

One vertex

Uniform rescaling of the boundary spins $j_f \to \lambda j_f$

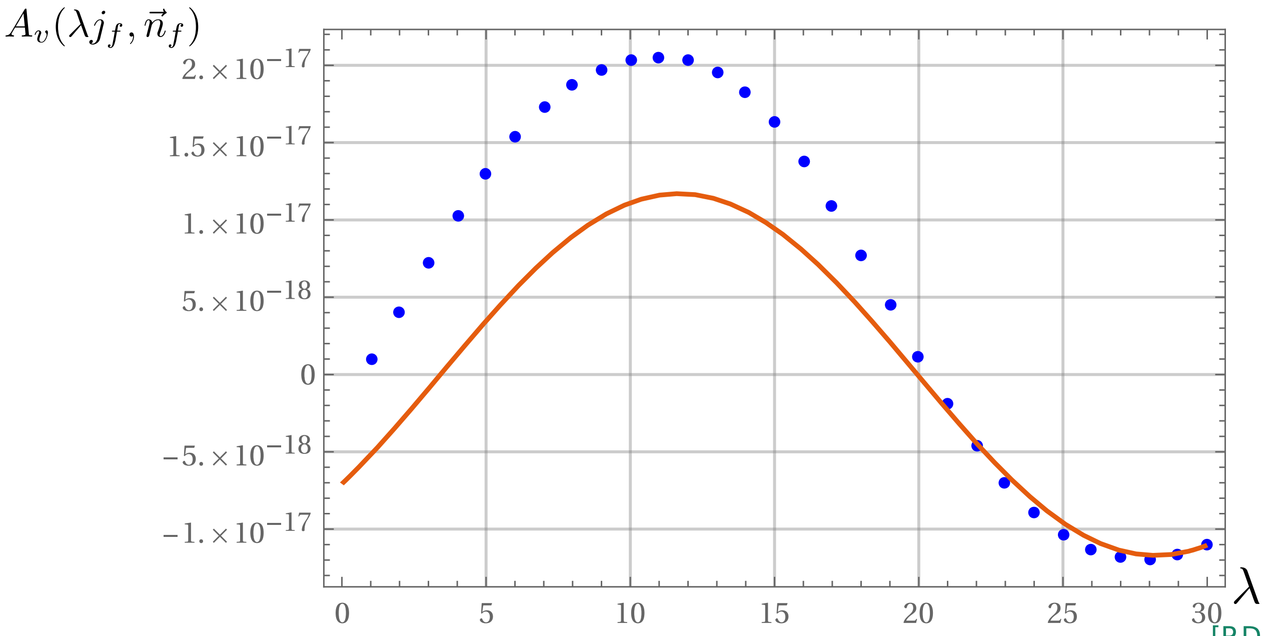
Boundary data representing Lorentzian 4-simplex (Livine-Speziale coherent states)

Exponentials of the Regge action (action for discrete GR)

Power law decreasing in the scale

$$A_v(\lambda j_f, \vec{n}_f) = \frac{1}{\lambda^{12}}(N_1 e^{i\lambda S_R} + N_2 e^{-i\lambda S_R}) + O(\lambda^{-13})$$

[J. Barrett and collaborators – 2010]



[F. Gozzini – 2021]

[P.D. M. Fanizza, G. Sarno, S. Speziale – 2019]

Connection with discrete GR

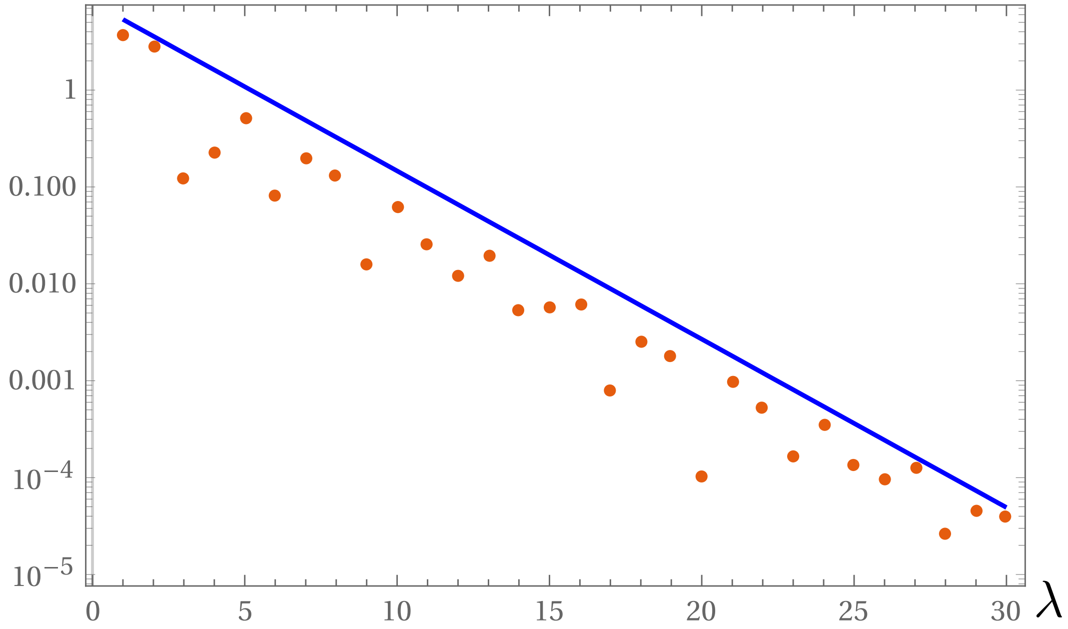
Many vertices

Classical equations of motions in the path integral emerge from stationary phase points equations (interference)

Is it the same for covariant LQG?

Solutions of classical equations of motions are stationary phase points

Only flat solutions of classical equations of motions exhibit stationary phase points behavior



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Problem?

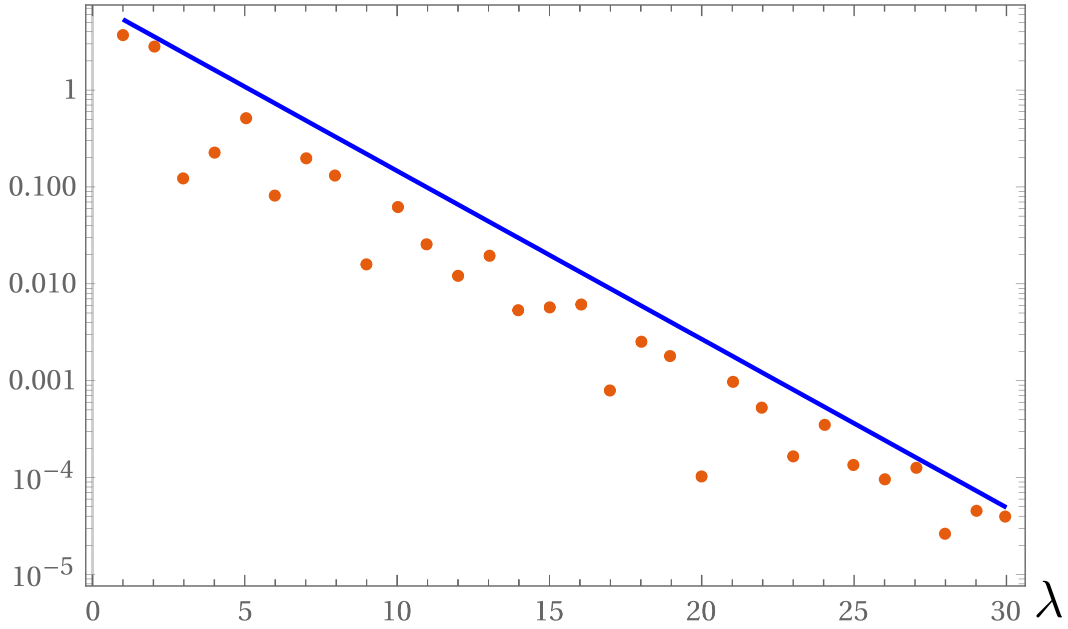
Promising action, different variables!

We have to change the theory!

Change the amplitude? (Better understanding on how the various ingredients mix)

Change the limit! Double scaling limit, how do we recover discrete Einstein equations?

$$j_f \gg 1 \quad \delta_f \ll 1 \quad j_f \delta_f = O(1)$$



My view on covariant loop quantum gravity

Theory is mature

UV finite transition amplitudes!

Dynamic of LQG with a path integral formulation

Very appealing geometrical interpretation

Excited about the numerical calculations (tool to point us in the right direction and highlight facts we have to explain)

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Applications

Black hole to white hole tunneling?

Correlations in the early universe?

[Many people, mainly at Western]

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Open questions

Do we really recover GR? (With the double scaling limit)

How do you couple matter properly? (Is the semiclassical limit compatible with the presence of matter?)

How do you remove the 2-complex regulator?

Do we need to renormalize the theory? (There are IR divergences)

Asking questions is a sign of a healthy theory!