Pietro Dona'

(*Fudan University*)

Yang-Mills condensate as dark energy and dark matter model

- a non-perturbative approach -

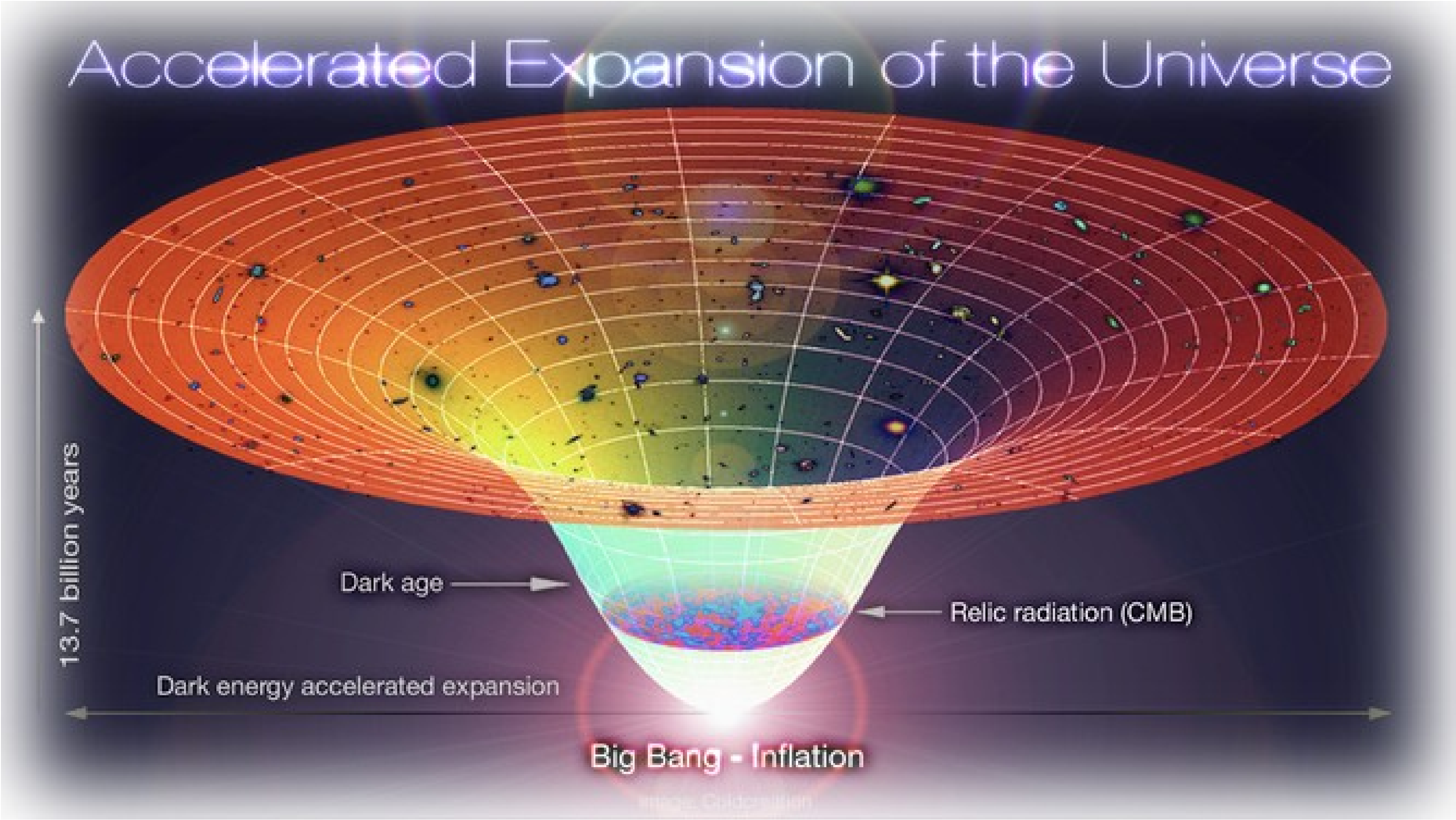
Phys. Rev. D 93, 044049; arXiv:1602.01772

14 - 4 - 2016

Fudan - Tokyo - Yonsei Workshop on Particle and Nuclear Physics

1) The Universe is undergoing an accelerated phase of expansion [Riess et al., Perlmutter et al. 1998]

- SN Ia dataset [Kowalski et al. 2008]
- CMB radiation [WMAP collaboration 2003]
- Large Scale Structures [2dFRGS, SDSS collaborations 2005/6]



Dark Energy

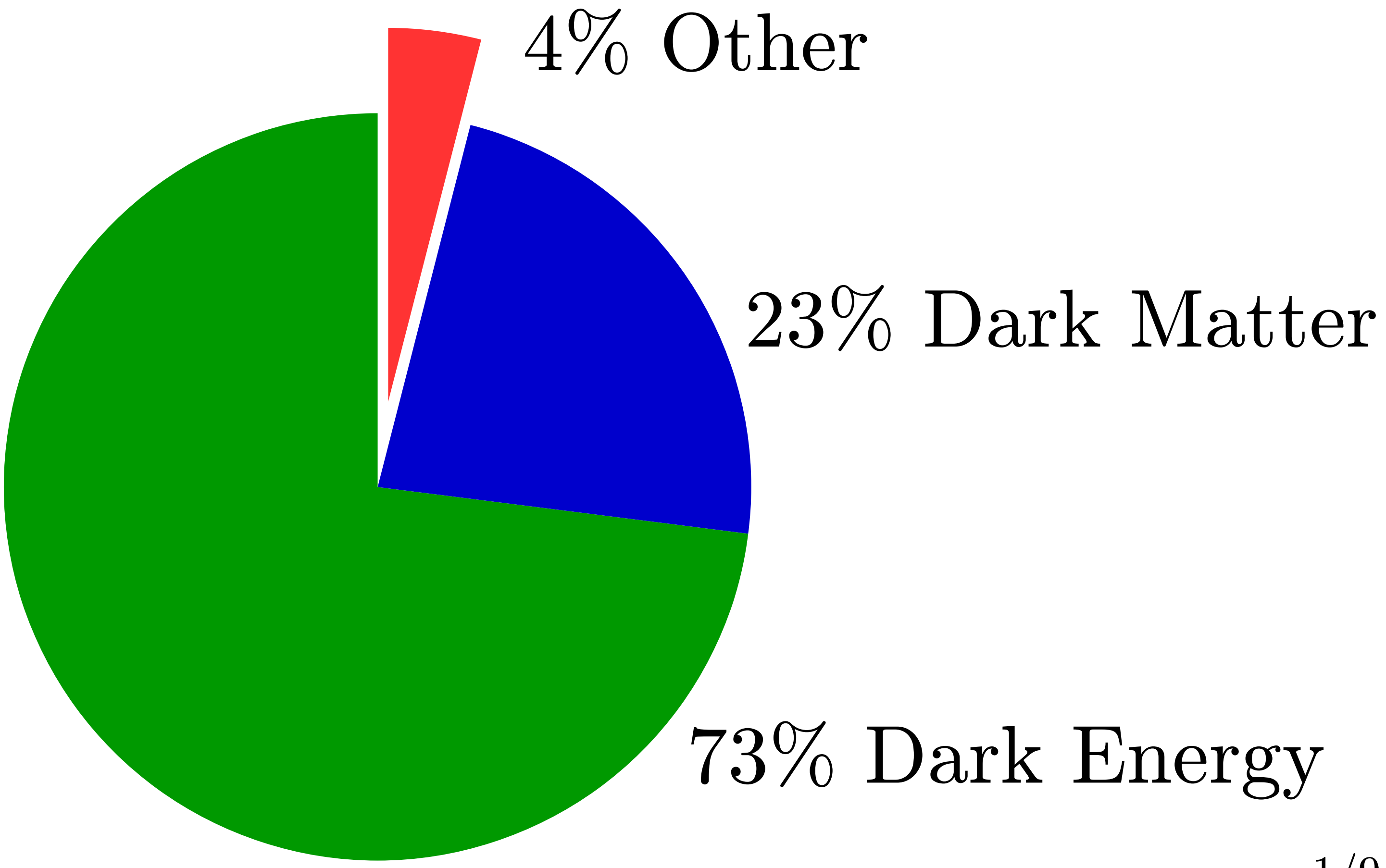
1) The Universe is undergoing an accelerated phase of expansion

2) Most part of the Universe is **dark**
[WMAP 7-years and Plank 2014]

Dark Energy

1) The Universe is undergoing an accelerated phase of expansion

2) Most part of the Universe is **dark**
[WMAP 7-years and Plank 2014]



Dark Energy

1) The Universe is undergoing an accelerated phase of expansion

2) Most part of the Universe is **dark**

3) Theoretical scenarios for Dark Energy

- $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$
- $T_{\mu\nu} = T_{\mu\nu}^{\text{Matter}} + T_{\mu\nu}^{\text{DarkEnergy}}$
- Modify General Relativity

Yang-Mills Condensate (YMC) as a DE model

[Y. Zhang PLB 1994;
Y. Zhang et al. PLB 2007 & JCAP 2008]

$$S = \int \sqrt{-g}\left[-\frac{R}{16\pi G} - \frac{1}{4g^2} F^a_{\mu\nu} F_a^{\mu\nu}\right]$$

Renormalization Group Improvement of the Quantum Effective Action

- RG improved Coleman-Weinberg potential

$$g \to g\left(\theta\right) \qquad\qquad \theta = -\frac{1}{2\kappa^2} F^a_{\mu\nu} F_a^{\mu\nu}$$

Yang-Mills Condensate (YMC) as a DE model

$$S = \int \sqrt{-g}\left[-\frac{R}{16\pi G} - \frac{1}{4g^2} F^a_{\mu\nu} F_a^{\mu\nu}\right]$$

Renormalization Group Improvement of the Quantum Effective Action

- RG improved Coleman-Weinberg potential
- Condensate phase of the YM field behaves like DE

→ naturally realize DE equation of state

→ evolving $w_{YMC} = \frac{1}{3} \rightsquigarrow w_{YMC} = -1$

→ the cosmic coincidence problem is avoided

→ interaction with dust, late time attractor solution

Yang-Mills Condensate (YMC) as a DE model

$$S = \int \sqrt{-g}\left[-\frac{R}{16\pi G} - \frac{1}{4g^2}F^a_{\mu\nu}F_a^{\mu\nu}\right]$$

Renormalization Group Improvement of the Quantum Effective Action

- RG improved Coleman-Weinberg potential
- Condensate phase of the YM field behaves like DE
- Perturbative analysis (one, two, three loops)

→ Stability of the perturbative computation?

→ DE energy scale is the IR (strong coupling)

We can answer this question by improving the model! → non-perturbative analysis

Requirements for YMC-DE model

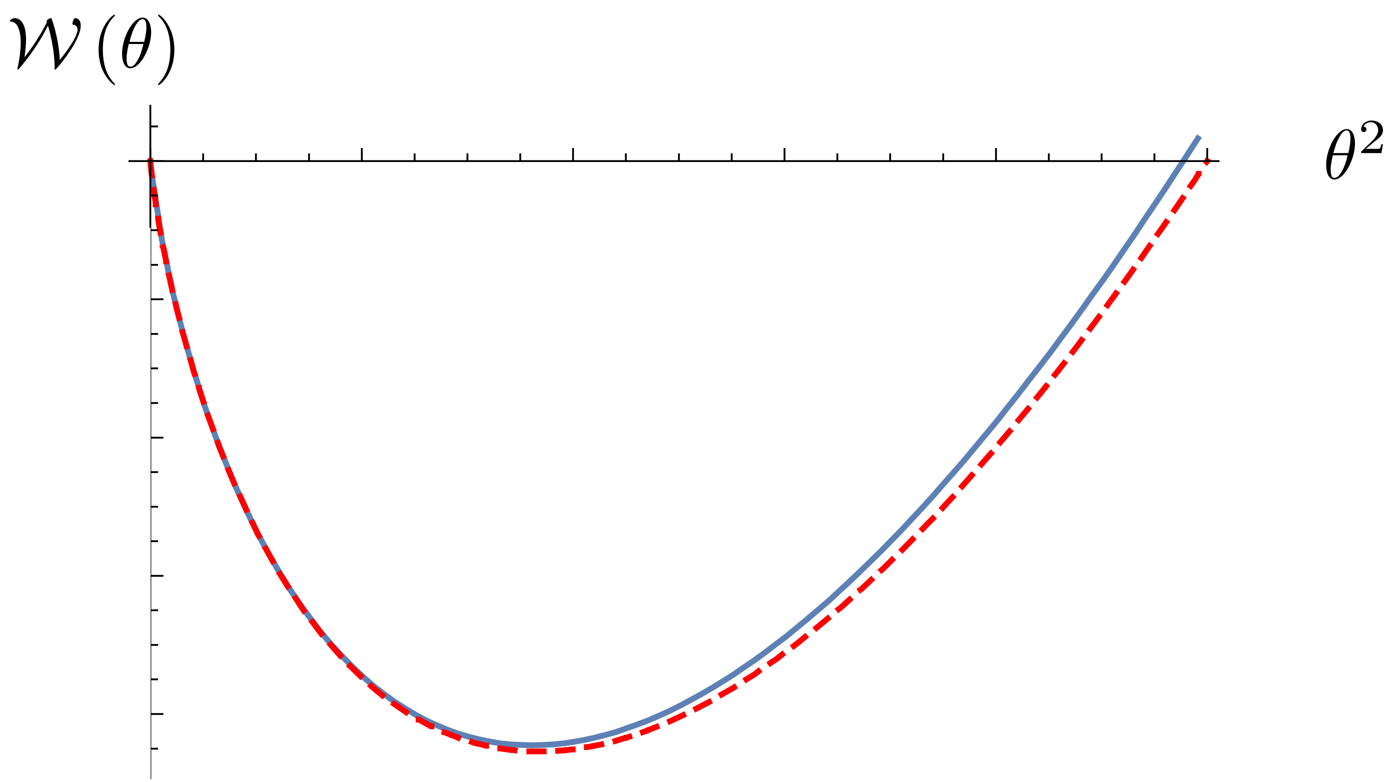
General assumptions on the Effective Lagrangian

1) possesses a non trivial minimum

2) reproduces the one-loop result in the right limit

3) linear to the bare Yang-Mills action in the UV

$$\mathcal{L}_{\text{eff}} = \mathcal{W}(\theta)$$



Consequences of the requirements

Are there general consequences?

- assume flat FLRW
- minimally coupled SU(2) YM fields
- compute stress-energy tensor, energy density and pressure, equation of state

$$A_0^a\,(t,\vec{x}) = 0,$$
$$A_i^a\,(t,\vec{x}) = \delta_i^a A\,(t)$$

$$\rho_{YMC} = -\mathcal{W}\,(\theta) + 2\mathcal{W}'\,(\theta)\,\theta \qquad p_{YMC} = \mathcal{W}\,(\theta) - \frac{2}{3}\mathcal{W}'\,(\theta)\,\theta$$

$$w_{YMC} \equiv \frac{p_{YMC}}{\rho_{YMC}} = -\frac{1-\frac{2}{3}\frac{\mathcal{W}'}{\mathcal{W}}\theta}{1-2\frac{\mathcal{W}'}{\mathcal{W}}\theta}$$

→ -1

→ 1/3

Features of the DE model

Friedmann Equations ⟶ Evolution Equations

$$\dot{\rho}_{YMC} + 3\frac{\dot{a}}{a}\left(\rho_{YMC} + p_{YMC}\right) = \xi_1 \rho_{YMC} + \xi_2 \rho_m$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = \xi_1 \rho_{YMC} + \xi_2 \rho_m$$

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}\left(\rho_r + p_r\right) = 0$$

Features of the DE model

Friedmann Equations ⟶ Evolution Equations

$$\dot{\rho}_{YMC} + 3\frac{\dot{a}}{a}\left(\rho_{YMC} + p_{YMC}\right) = \xi_1 \rho_{YMC} + \xi_2 \rho_m$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = \xi_1 \rho_{YMC} + \xi_2 \rho_m$$

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}\left(\rho_r + p_r\right) = 0$$

$\xi_1 = \xi_2 = 0$ Free case

$$\dot{\theta}\left(\mathcal{W}' + 2\mathcal{W}''\theta\right) + 4\frac{\dot{a}}{a}\mathcal{W}\theta = 0 \longrightarrow \sqrt{\theta}\mathcal{W}'\left(\theta\right) = \alpha a^{-2}$$

Features of the DE model

Friedmann Equations ⟶ Evolution Equations

$$\dot{\rho}_{YMC} + 3\frac{\dot{a}}{a}\left(\rho_{YMC} + p_{YMC}\right) = \xi_1 \rho_{YMC} + \xi_2 \rho_m$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = \xi_1 \rho_{YMC} + \xi_2 \rho_m$$

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}\left(\rho_r + p_r\right) = 0$$

✓ $\xi_1 = \xi_2 = 0$ Free case

$\xi_1 = 0\,,\ \xi_2 \neq 0$ Interaction proportional to matter energy density

$\rho_m = a^{\xi_2 - 3}$ No stable solutions

Features of the DE model

Friedmann Equations ⟶ Evolution Equations

$$\dot{\rho}_{YMC} + 3\frac{\dot{a}}{a}\left(\rho_{YMC} + p_{YMC}\right) = \xi_1 \rho_{YMC} + \xi_2 \rho_m$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = \xi_1 \rho_{YMC} + \xi_2 \rho_m$$

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}\left(\rho_r + p_r\right) = 0$$

✓ $\xi_1 = \xi_2 = 0$ Free case

✗ $\xi_1 = 0\,,\ \xi_2 \neq 0$ Interaction proportional to matter energy density

$\xi_1 \neq 0\,,\ \xi_2 \neq 0$ Interaction proportional to both energy densities

Features of the DE model

Friedmann Equations ⟶ Evolution Equations

$$\dot{\rho}_{YMC} + 3\frac{\dot{a}}{a}(\rho_{YMC} + p_{YMC}) = \xi_1 \rho_{YMC} + \xi_2 \rho_m$$

$$\dot{\rho}_m + 3\frac{\dot{a}}{a}\rho_m = \xi_1 \rho_{YMC} + \xi_2 \rho_m$$

$$\dot{\rho}_r + 3\frac{\dot{a}}{a}(\rho_r + p_r) = 0$$

✔ $\xi_1 = \xi_2 = 0$ Free case

✘ $\xi_1 = 0\,,\ \xi_2 \neq 0$ Interaction proportional to matter energy density

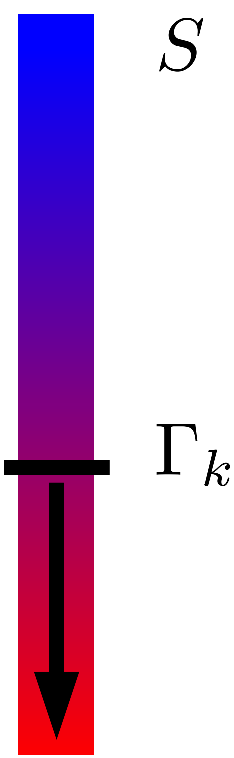
✘ $\xi_1 \neq 0\,,\ \xi_2 \neq 0$ Interaction proportional to both energy densities

$\xi_1 \neq 0\,,\ \xi_2 = 0$ Interaction proportional to YMC energy density

? look for late time attractor solutions for a specific effective action

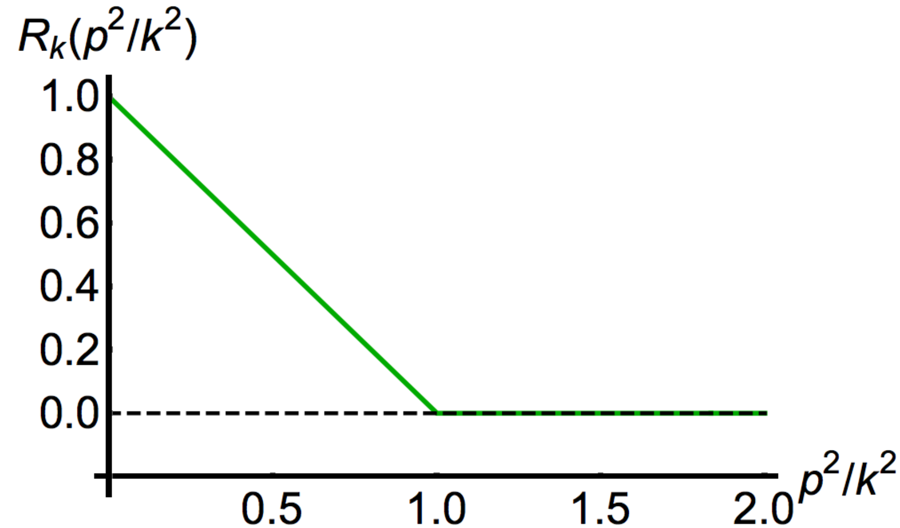
Functional Renormalization Group

A tool to study QFTs and Statistical Systems in the non-perturbative regime



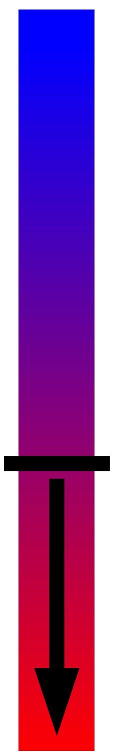
- Wilsonian momentum-shell wise integration of the path-integral
- Mass-like regulator: suppresses quantum fluctuations with momenta lower than an IR momentum cutoff scale.

$$\int_x \frac{1}{2}\phi(x)R_k(\Box)\phi(x)$$



Functional Renormalization Group

A tool to study QFTs and Statistical Systems in the non-perturbative regime



S

- Wilsonian momentum-shell wise integration of the path-integral
- mass-like regulator: suppresses quantum fluctuations with momenta lower than an IR momentum cutoff scale.

Γ_k

- scale-dependent effective action Γ_k, contains the effect of quantum fluctuations with momenta k
- it obey the following exact equation

$$k\partial_k \Gamma_k = \frac{1}{2} \mathrm{STr} \left[\left(\Gamma_k^{(2)} + R_k \right)^{-1} k\partial_k R_k \right]$$

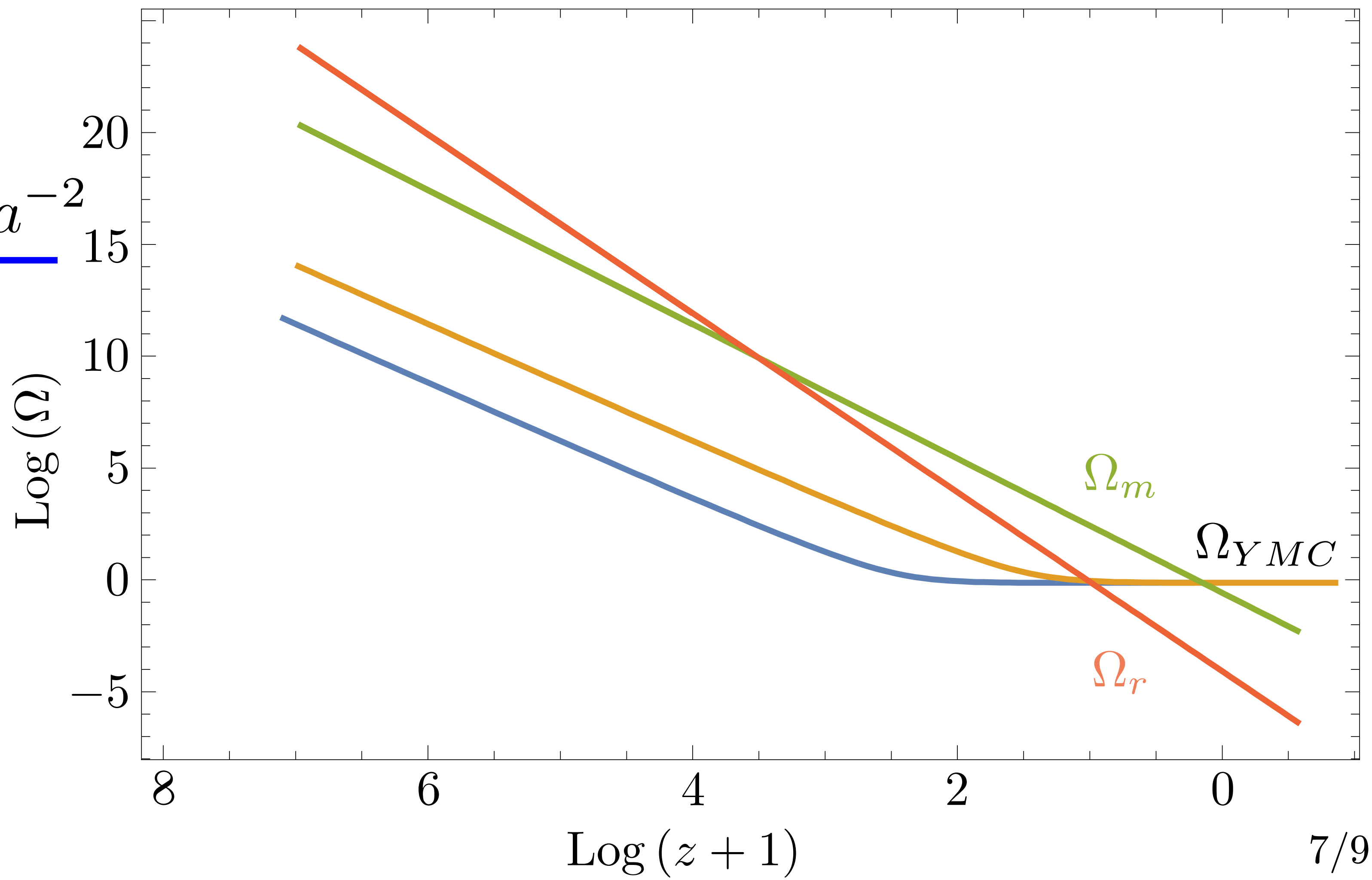
FRG - DE model

$$\mathcal{W}(\theta) = \frac{g^2\theta}{2\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s} e^{-s\left(\frac{k^4}{g^2\theta}\right)^{1/2}} \left(\frac{1}{4\sinh^2(s)} + 1 - \frac{1}{4s^2}\right)$$

$\xi_1 = \xi_2 = 0$

Free case

$$\sqrt{\theta}\mathcal{W}'(\theta) = \alpha a^{-2}$$



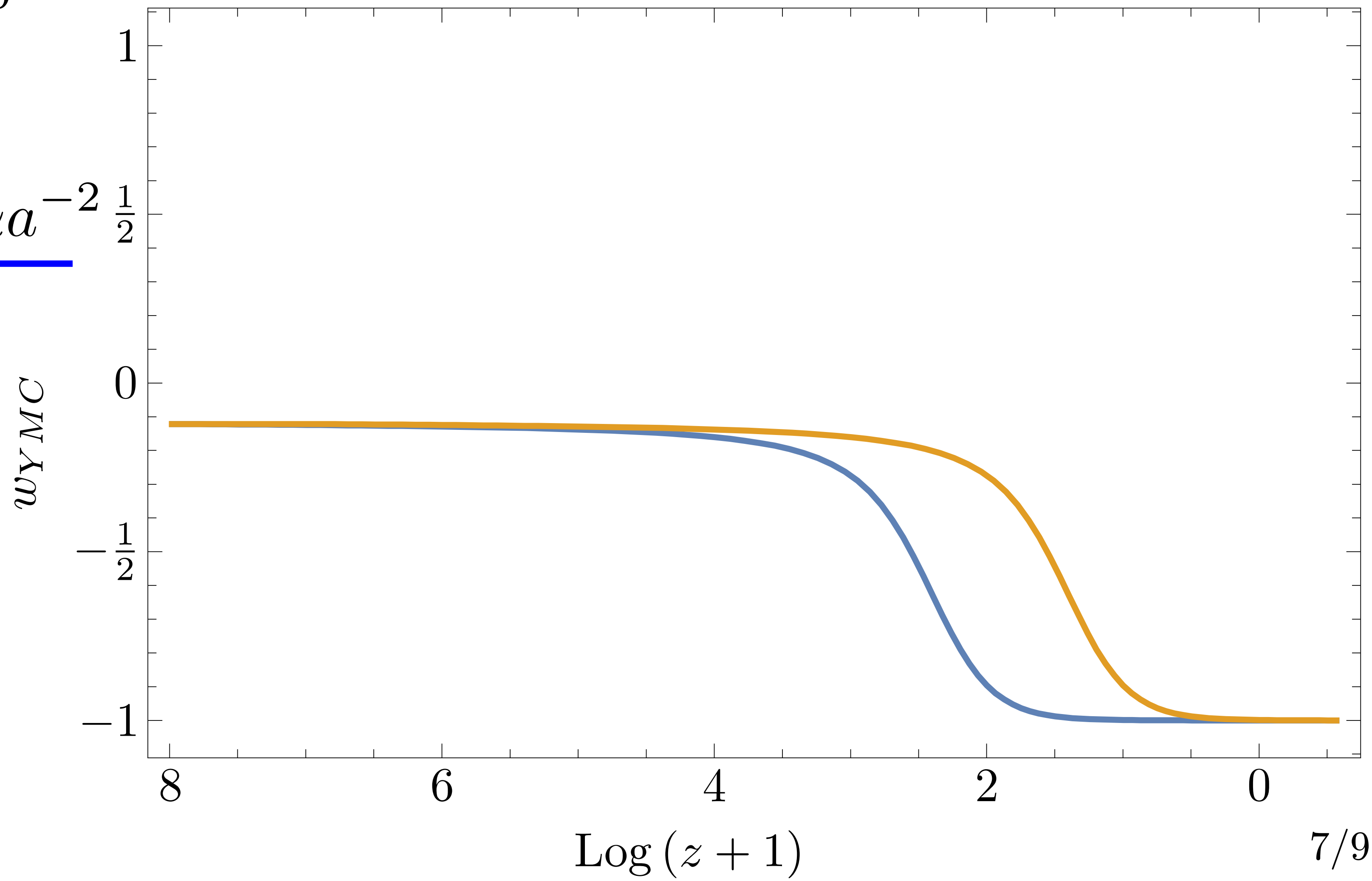
FRG - DE model

$$\mathcal{W}\left(\theta\right) = \frac{g^2\theta}{2\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s} e^{-s\left(\frac{k^4}{g^2\theta}\right)^{1/2}}\left(\frac{1}{4\sinh^2\left(s\right)} + 1 - \frac{1}{4s^2}\right)$$

✓ $\xi_1 = \xi_2 = 0$

Free case

$\sqrt{\theta}\mathcal{W}'\left(\theta\right) = \alpha a^{-2}$



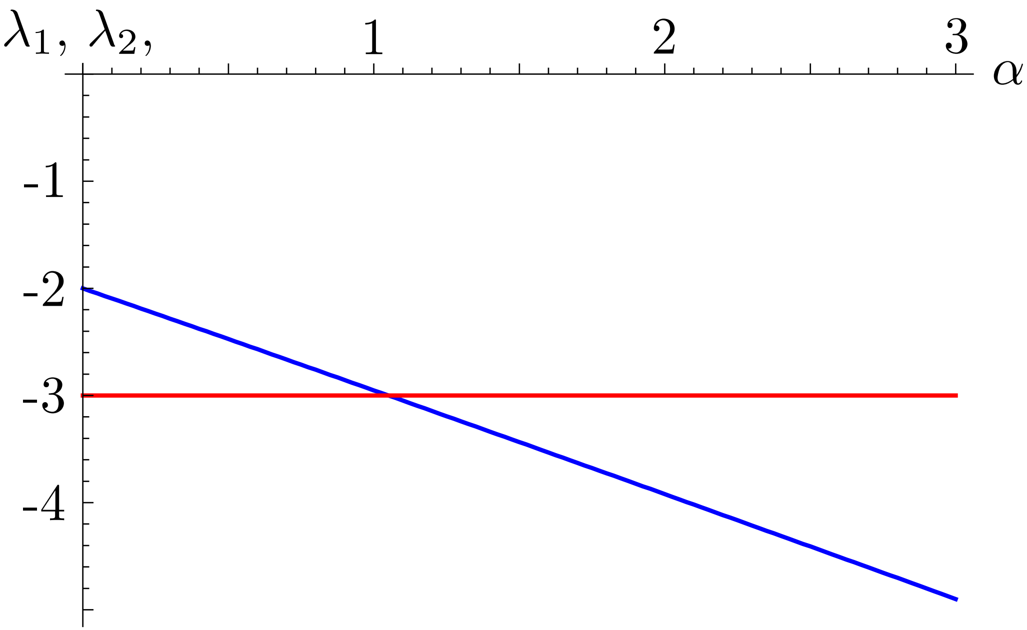
FRG - DE model

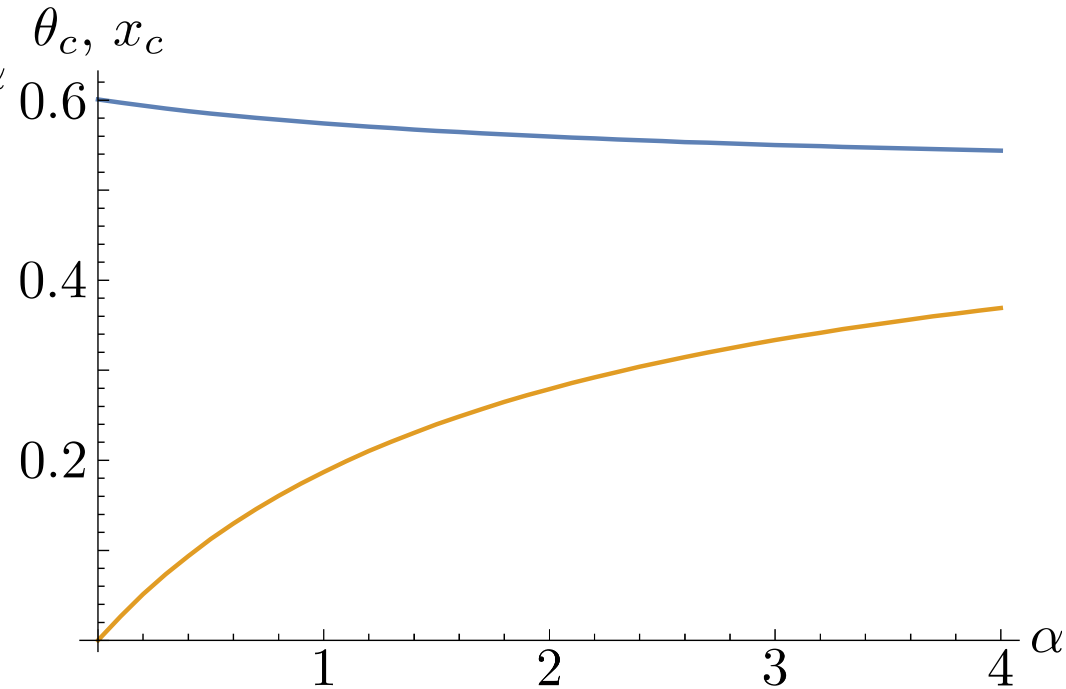
$$\mathcal{W}(\theta) = \frac{g^2\theta}{2\pi^2}\int_0^\infty \frac{\mathrm{d}s}{s} e^{-s\left(\frac{k^4}{g^2\theta}\right)^{1/2}} \left(\frac{1}{4\sinh^2(s)} + 1 - \frac{1}{4s^2}\right)$$

$\xi_1 \neq 0\,,\ \xi_2 = 0$

Late time attractor solutions search

$$\theta'\left(\mathcal{W}' + 2\theta\mathcal{W}''\right) + 4\theta\mathcal{W}' = +\alpha\left(\mathcal{W} - 2\theta\mathcal{W}'\theta + x\right)$$
$$x' + 3x = -\alpha\left(\mathcal{W} - 2\theta\mathcal{W}'\theta + x\right)$$





Conversion of DE-YMC into DM

- QCD invisible axion [Peccei-Quinn 1977]
  (associated to the solution of the CP problem)

  → $SM \times SU(2)_D \times U(1)_{PQ}$

  → Goldstone boson of the spontaneously broken global axial symmetry

  → good candidate for cold and hot DM

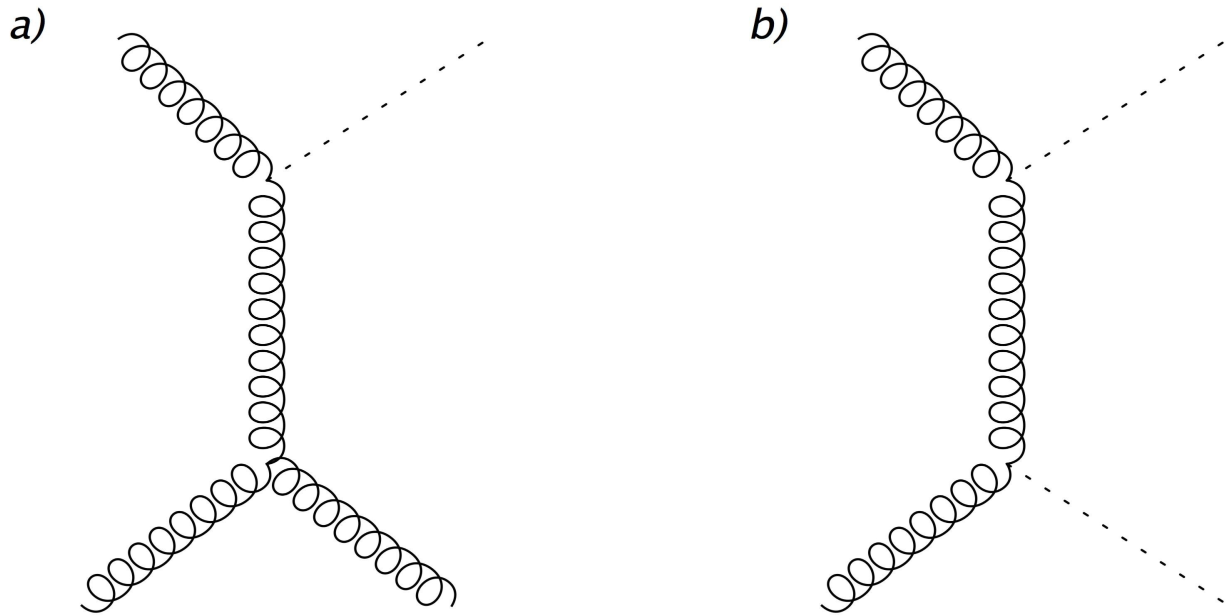
Conversion of DE-YMC into DM

- QCD invisible axion [Peccei-Quinn 1977]
  (associated to the solution of the CP problem)

- Coupled to the dark-gauge sector $\mathcal{O}_{aF\tilde{F}} = \frac{a}{\mathcal{M}} F^{a}_{\mu\nu} \tilde{F}^{\mu\nu}_{a}$

  → Cubic term does not spoil the effective Lagrangian of the YMC

  → portal between DE and DM



Conversion of DE-YMC into DM

- QCD invisible axion [Peccei-Quinn 1977]
  (associated to the solution of the CP problem)
- Coupled to the dark-gauge sector $\mathcal{O}_{aF\tilde{F}} = \frac{a}{\mathcal{M}} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$
- Estimate the decay rate

$$\tau \sim \left(\frac{\mathcal{M}}{GeV} \right)^2 Gyr$$

10% variation of DM can be visible in cosmological time (10 Gyr) for the parameter sufficiently large $\mathcal{M} \simeq 120 GeV$

Conclusions

- Under very general assumptions a YMC originates for SU(2) YM
- YMC as a working Dark Energy model
- Explicit computation non-perturbative techniques (FRG approach)
- QCD axions can be emitted by the YMC in cosmological time, converting DE density into cold DM energy density.

Conclusions

- Under very general assumptions a YMC originates for SU(2) YM
- YMC as a working Dark Energy model
- Explicit computation non-perturbative techniques (FRG approach)
- QCD axions can be emitted by the YMC in cosmological time, converting DE density into cold DM energy density.

Thank you for your attention!