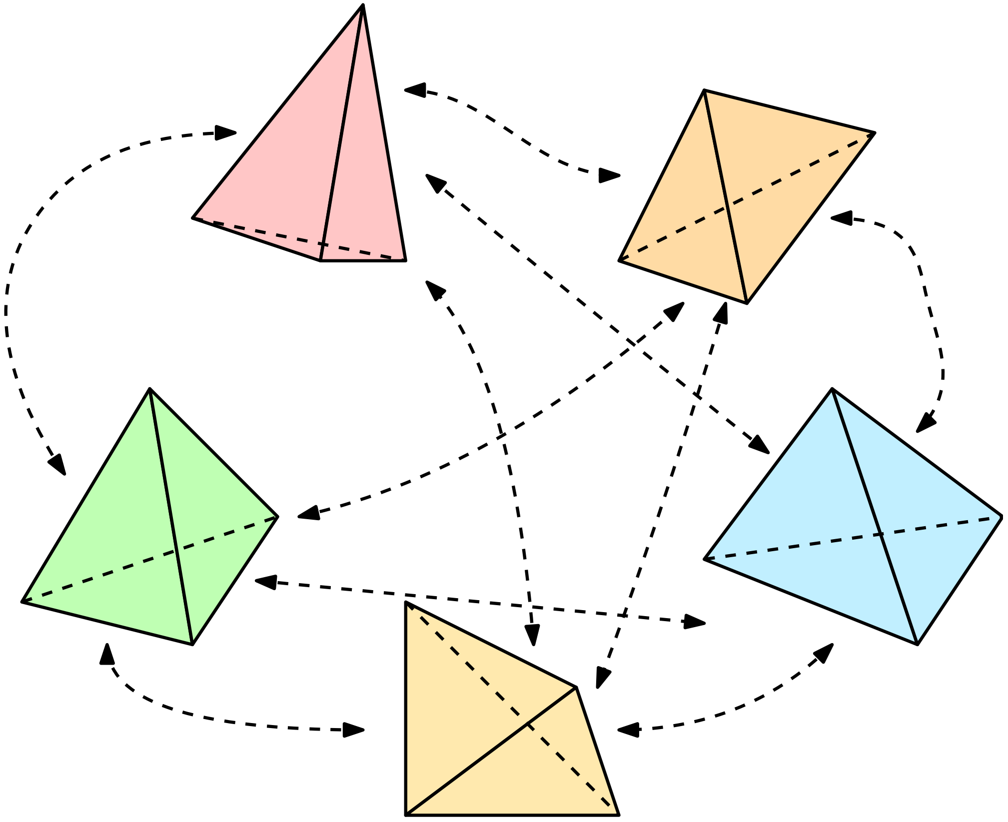
Pietro Donà



Local flatness in spin foam theories

EPRL spin foam theory

- Dynamics for LQG as a path integral
- Connection with discrete GR

EPRL spin foam theory

- Dynamics for LQG as a path integral
- Connection with discrete GR
- Recover GR in the semiclassical limit? (for real)
- Why can we reconstruct Regge geometries (locally=vertex by vertex)?
- Where are Einstein equations?

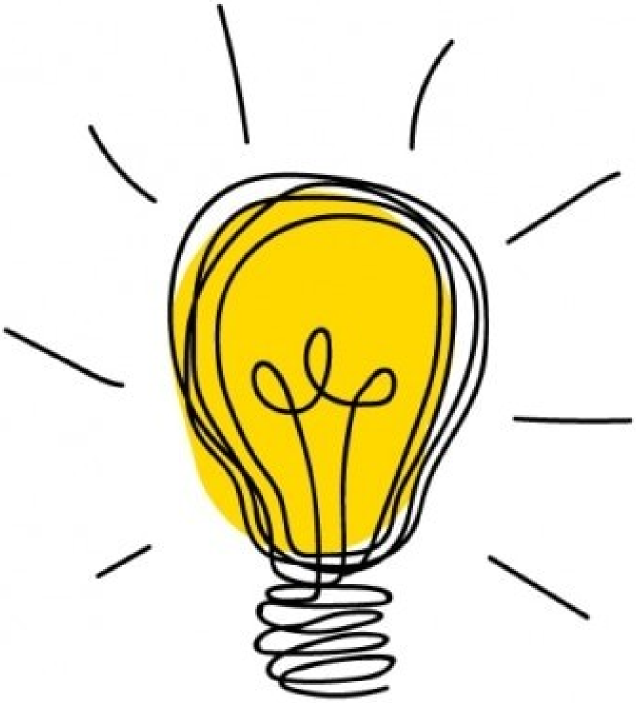
Regge geometries in spin foam models

Flat 4-simplices (lorentzian and euclidean) emerge from the large quantum numbers analysis of the spin foam vertices (local), the holonomies and the action are also Regge-like

Different derivations, similar conclusion, physically different models
(Topological BF SU(2), EPRL Lorentzian and Euclidean, EPRL extensions, Barrett-Crane)

Geometry appears always in similar ways. There are lots of papers explaining how.

Why?



The local emergence of Regge geometries is a consequence of an often overlooked property...

Local flatness

general discussion, model independent!

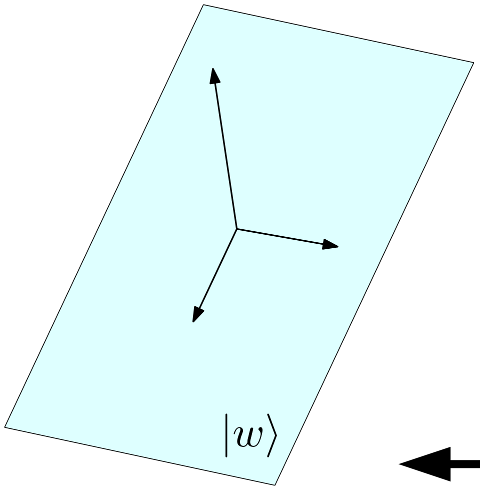
Holonomies as map between framed planes

$$g = e^{\frac{\omega}{2}}|w\rangle\langle z| + e^{-\frac{\omega}{2}}|w][z|$$

(norm 1 spinors, redundant, two extra phases)

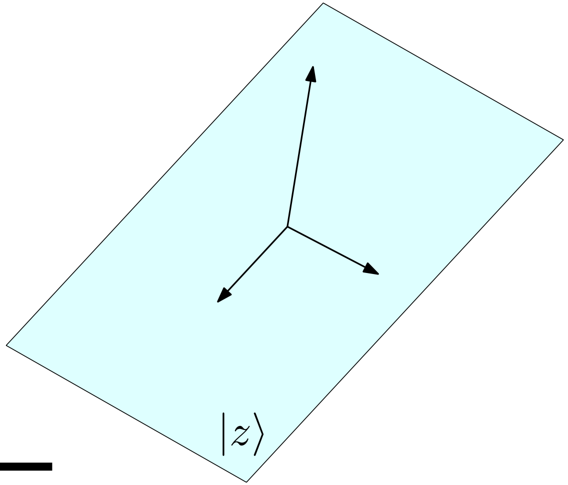
Parametrization: a source spinor, a target spinor and a complex number

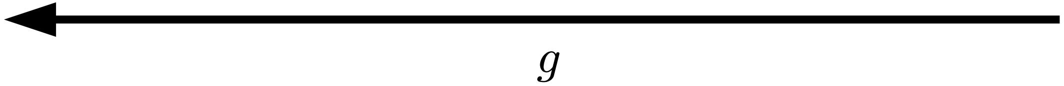
(heavily inspired by twisted geometries literature)



$\mathrm{Re}\ \omega$ Represent a boost between framed planes (polar decomposition)

$\mathrm{Im}\ \omega$ Represent a twist of the frames (misalignment)

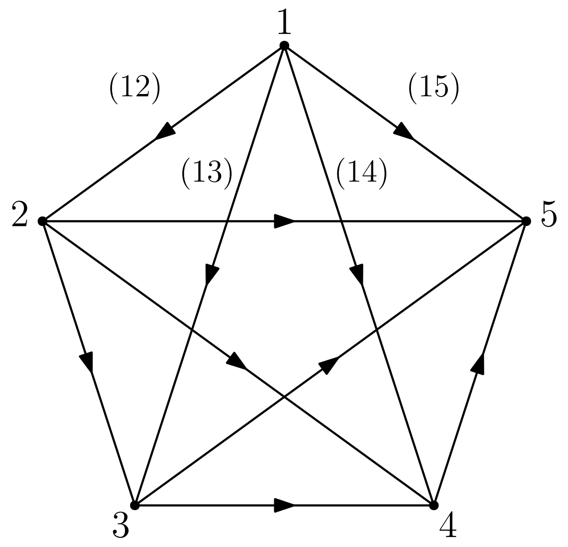




(or if you prefer a triangle with a frame)

Setting the scene for local flatness

Consider the 2-complex of one 4-simplex (no geometric data, just combinatorial information)



Variables: Ten $SL(2,\mathbb{C})$ holonomies describe the parallel transport from one edge to another.
(four framed planes to each edge and a complex number to each face)

Request: each 4-simplex is flat!
(a general 2-complex made of such vertices is locally flat).

$$g_{ca}g_{bc}g_{ab} = \mathbb{1}$$

Holonomies and local flatness

Local flatness

$$g_{ca}g_{bc}g_{ab} = \mathbb{1}$$

$=$

Complex angles determined by spinors

$$\cosh(\omega_{ab} + i\xi^c_{ab}) = \cos\hat{\theta}^c_{ab}$$

$$\sin\phi^b_{ac}\sinh(\omega_{ab} + i\xi^c_{ab}) = \sin\phi^c_{ab}\sinh(\omega_{ca} + i\xi^b_{ac})$$

4D dihedral angles defined using spherical cosine laws (exist a local embedding in 4D)

$$\cos\hat{\theta}^c_{ab} = \frac{-|\langle z_{ca}|z_{cb}]|^2 + |\langle z_{ab}|z_{ac}\rangle|^2|\langle z_{ba}|z_{bc}\rangle|^2 + |\langle z_{ac}|z_{ab}]|^2|\langle z_{ba}|z_{bc}]|^2}{2|\langle z_{ac}|z_{ab}\rangle\langle z_{ac}|z_{ab}]\langle z_{ba}|z_{bc}\rangle\langle z_{ba}|z_{bc}]|} = \frac{\cos\phi^c_{ab} + \cos\phi^a_{bc}\cos\phi^b_{ac}}{\sin\phi^a_{bc}\sin\phi^b_{ac}}$$

Edge dependent twist angle (a SU(2) invariant way to measure the twist between frames)

$$\xi^c_{ab} = \arg\left(\frac{\langle z_{ac}|z_{ab}]\langle z_{ab}|z_{ac}\rangle}{\langle z_{bc}|z_{ba}\rangle[z_{ba}|z_{bc}\rangle}\right)$$

Holonomies and local flatness

Local flatness

$$g_{ca}g_{bc}g_{ab} = \mathbb{1}$$

=

Complex angles determined by spinors

$$\cosh(\omega_{ab} + i\xi^c_{ab}) = \cos\hat{\theta}^c_{ab}$$

$$\sin\phi^b_{ac}\sinh(\omega_{ab} + i\xi^c_{ab}) = \sin\phi^c_{ab}\sinh(\omega_{ca} + i\xi^b_{ac})$$

Solutions : complex angle in terms of geometric angles

$$\omega_{ab} = i\epsilon\hat{\theta}^c_{ab} - i\xi^c_{ab}$$

Independence from the choice of cycle!

Lorentzian sector $\quad |\cos\hat{\theta}^c_{ab}| > 1$

$$\omega_{ab} = \epsilon\theta_{ab} + i\epsilon\chi_{ab}\pi - i\xi_{ab}$$

Holonomies and local flatness

Local flatness

$$g_{ca}g_{bc}g_{ab} = \mathbb{1}$$

$=$

Complex angles determined by spinors

$+$

“shape” matching conditions (strongly)

$+$ closure conditions

(gives areas up to a scale, make the statement stronger but simpler)

In ALL locally flat Lorentzian spin foam models shape matched Lorentzian 4-simplices emerge!

Classical! No amplitude, embedding map, semiclassical regime or critical point eqs.

Local flatness in the EPRL model

(all the spin foam models are locally flat)

$$A_v = \int \prod_a \mathrm{d}g_a \delta(g_1) \prod_{ab} D^{(\gamma j_{ab}, j_{ab})}_{j_{ab}\zeta_{ba} j_{ab}\zeta_{ab}} \left(\boxed{g_b^{-1} g_a}\right)$$

(edge holonomies)

The EPRL model is locally flat (imposed strongly)

$$A_v = \int \left(\prod_{ab} \mathrm{d}g_{ab} D^{(\gamma j_{ab}, j_{ab})}_{j_{ab}\zeta_{ba} j_{ab}\zeta_{ab}} \left(\boxed{g_{ab}}\right) \right) \mathcal{C}_{LF}(g_{ab}, \cdots, g_{cd})$$

(wedge holonomies)

$$+$$

$$\begin{aligned} \mathcal{C}_{LF}(g_{ab}, \cdots, g_{cd}) = & \delta\left(g_{13}^{-1} g_{23} g_{12}\right) \delta\left(g_{14}^{-1} g_{24} g_{12}\right) \delta\left(g_{15}^{-1} g_{25} g_{12}\right) \cdot \\ & \delta\left(g_{14}^{-1} g_{34} g_{13}\right) \delta\left(g_{15}^{-1} g_{35} g_{13}\right) \delta\left(g_{15}^{-1} g_{45} g_{14}\right) \ . \end{aligned}$$

What are the spin foam models telling us? (or should)

Connection with LQG → Using unit-irrep of the Lorentz group

Closure conditions
(consequence of the SU(2) invariance of the amplitude) → Framed planes at the same edge close forming a framed tetrahedron (angles given by areas/spins)

Alignment equations
(how to “glue” vertices) → The spinors parameterizing the holonomies match between vertices/with the boundary ones

Discrete Einstein equations
(when we consider closed faces) → Which conditions the holonomy around close faces satisfy? (e.g. flatness for topological models)

What is the EPRL model telling us?

Connection with LQG → Quantum polyhedra/intertwiners in the large spins regime

Closure conditions
(consequence of the SU(2) invariance of the amplitude) → Use LQG intertwiners, closure = invariance

Alignment equations
(how to “glue” vertices) → Holonomies in different vertices glues trivially. Same spinors. Simplicity constraints

Discrete Einstein equations
(when we consider closed faces) → Flatness problems. Not satisfactory workarounds, does not provide a mechanism to obtain the Regge equations

Conclusions

Local flatness is responsible of the local emergence of Regge geometry in spin foam models

Integrating over locally flat holonomies with $\mathrm{SU}(2)$ edge invariance restricted on the Lorentzian sector = Summing over Lorentzian 4-simplices (connection with effective spin foam models and area-angle Regge calculus)

Shape matching imposed strongly on the intrinsic classical geometry! (not connected with the action of the model)

Conclusions

Then what? What do we do with this?

Conclusions

I do not know!

Understanding what is coming from what is fundamental!

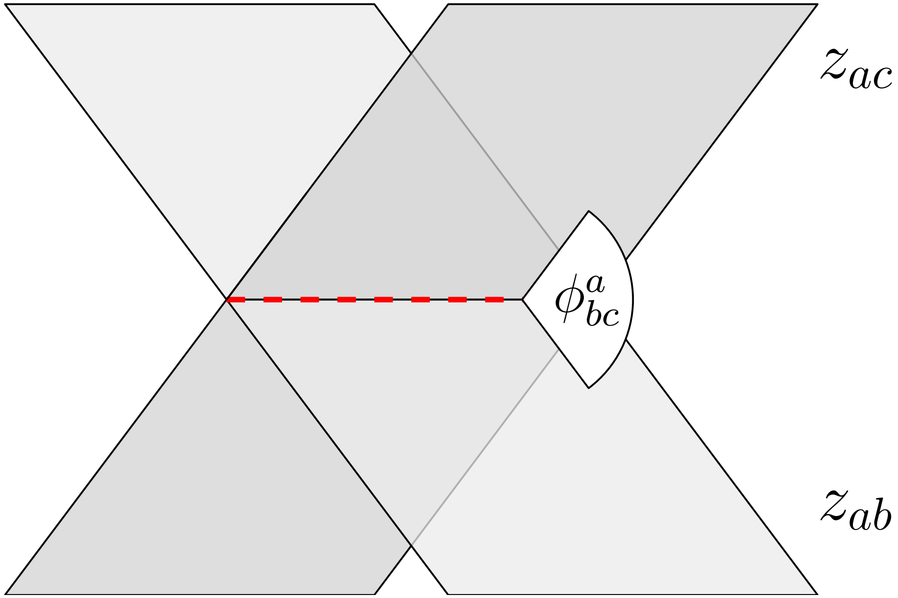
Let’s start with a shopping list of ingredients we want!

Some geometric quantities

3d dihedral angles

(angle between 2 framed planes at the same edge)

$$\cos\phi^a_{bc} = \vec{n}_{ab}\cdot\vec{n}_{ac} = 2|\langle z_{ab}|z_{ac}\rangle|^2 - 1$$



Some geometric quantities

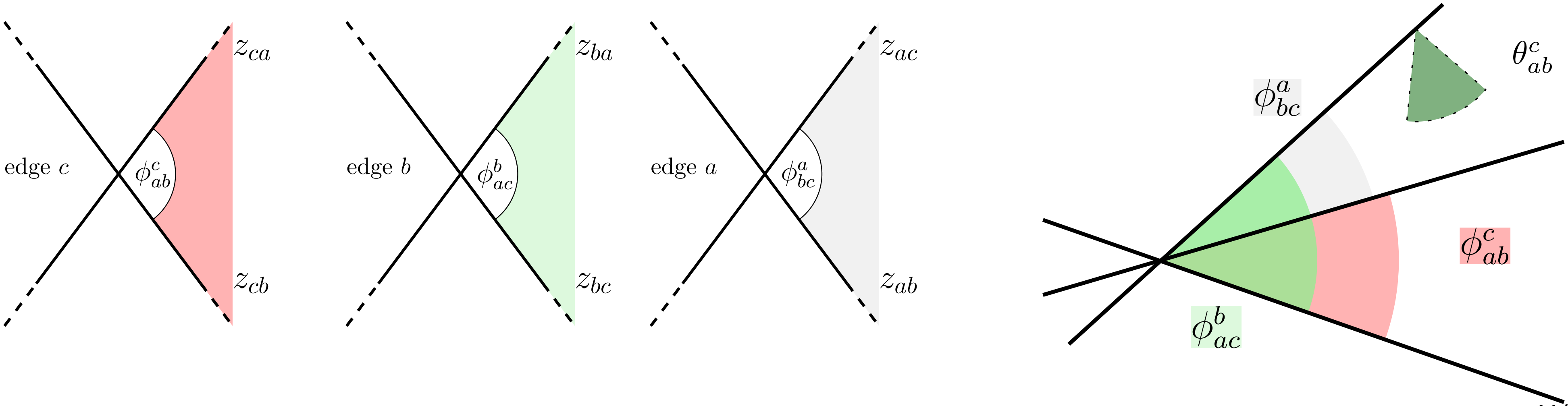
Spherical cosine law and sine law

(local embedding of 3D hyperplanes in 4D – signature?)

$$\cos\hat{\theta}^c_{ab} = \frac{-|\langle z_{ca}|z_{cb}]|^2 + |\langle z_{ab}|z_{ac}\rangle|^2|\langle z_{ba}|z_{bc}\rangle|^2 + |\langle z_{ac}|z_{ab}]|^2|\langle z_{ba}|z_{bc}]|^2}{2|\langle z_{ac}|z_{ab}\rangle\langle z_{ac}|z_{ab}]\langle z_{ba}|z_{bc}\rangle\langle z_{ba}|z_{bc}]|} = \frac{\cos\phi^c_{ab} + \cos\phi^a_{bc}\cos\phi^b_{ac}}{\sin\phi^a_{bc}\sin\phi^b_{ac}}$$

$$\sin\phi^b_{ac}\sinh\hat{\theta}^c_{ab} = \sin\phi^c_{ab}\sinh\hat{\theta}^b_{ac}$$

(schematic picture in 1 lower dimension – I cannot draw in 4D)



Some geometric quantities

Twist angle

(measure twist between frames using a third as reference -
the same one defined by Bianca and Jimmy or Simone and Fabio)

$$\xi^c_{ab} = \arg\left(\frac{\langle z_{ac}|z_{ab}]\langle z_{ab}|z_{ac}\rangle}{\langle z_{bc}|z_{ba}\rangle[z_{ba}|z_{bc}\rangle}\right)$$