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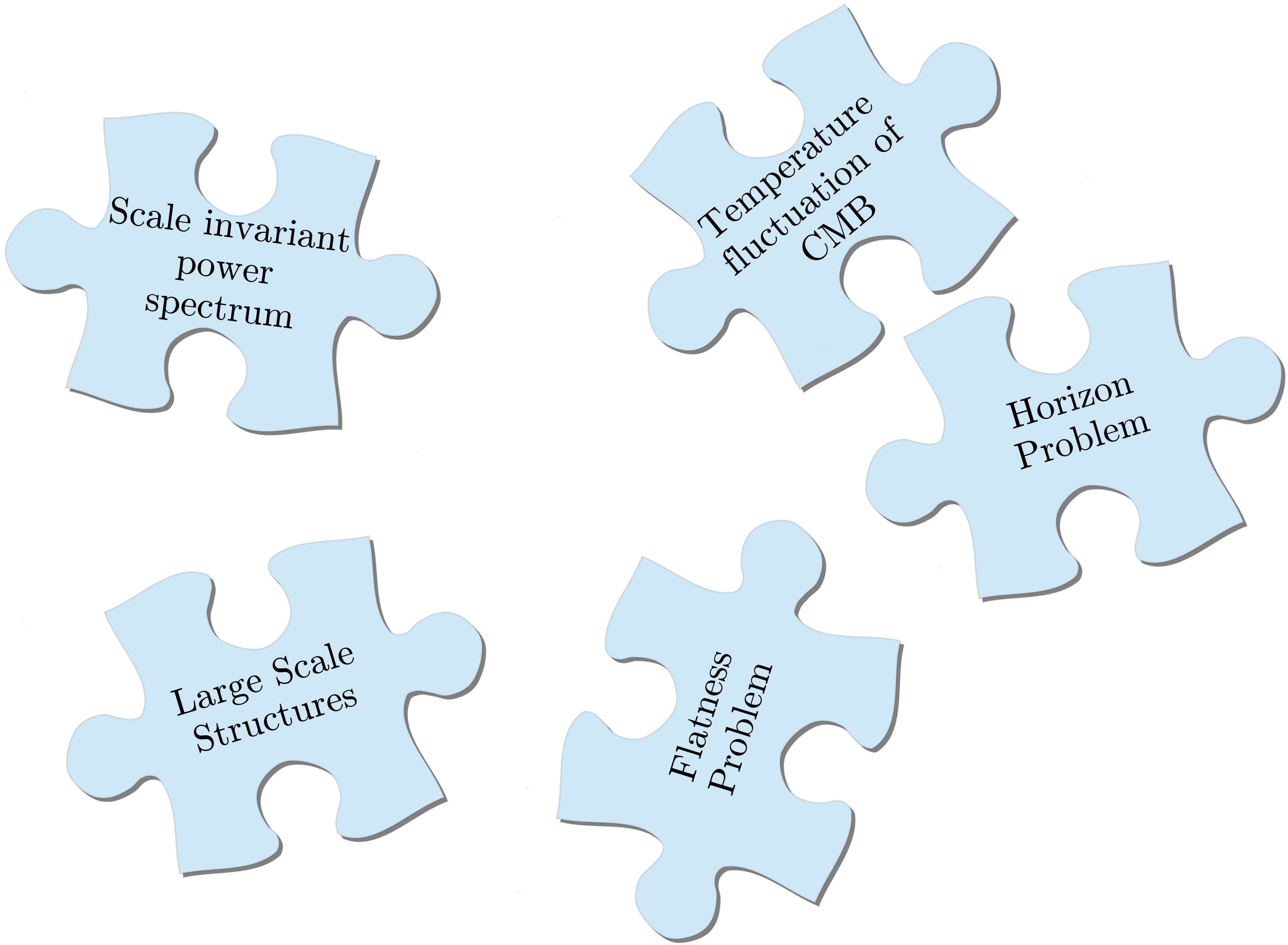
Cosmological applications of coherent states

16th January 2016

Based on:
PD, Antonino Marciano' - arXiv:1605.09337 (10.1103/PhysRevD.94.123517)
Suddhasattwa Brahma, PD, Antonino Marciano' - arXiv:1612.00760

The Big Bang puzzle



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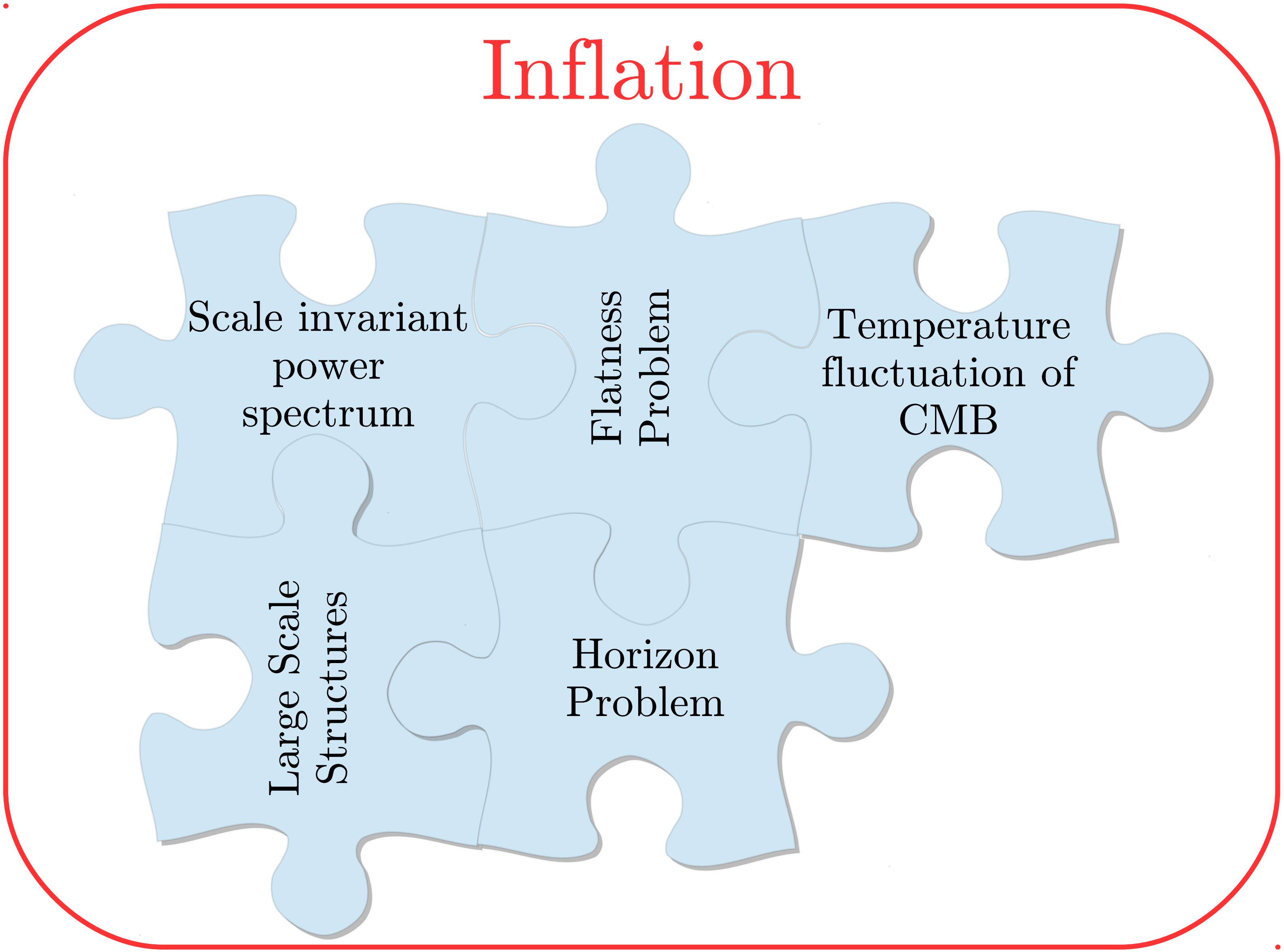
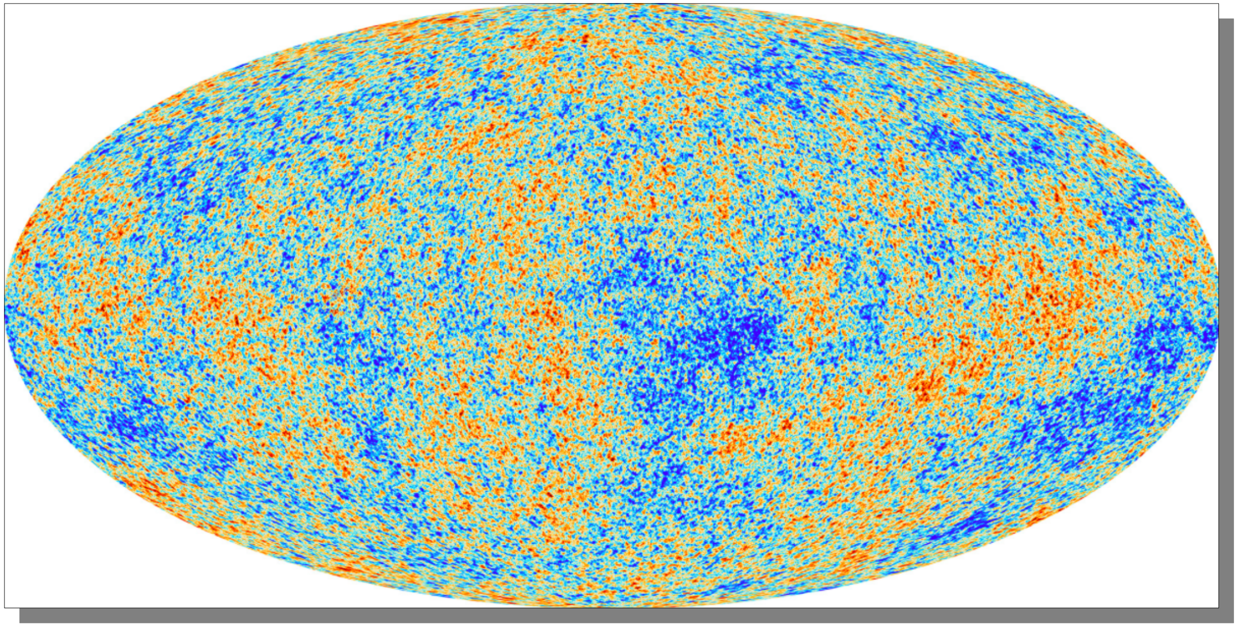


Inflation Recipe

Classical scalar field in FLRW background

very good

Inflation Recipe

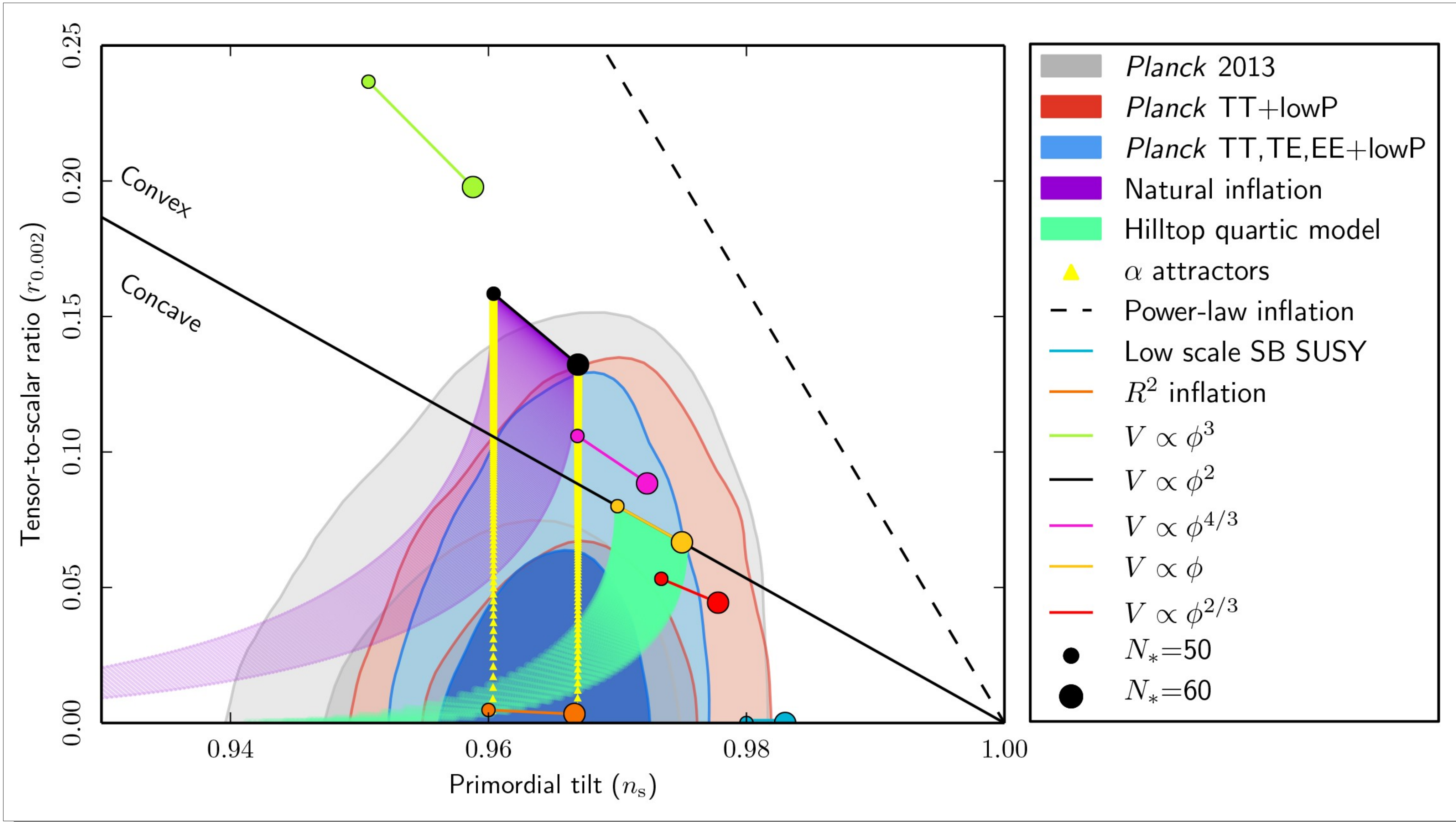


Classical scalar field in FLRW background

Local isotropy and homogeneity (CMB, galaxy count)

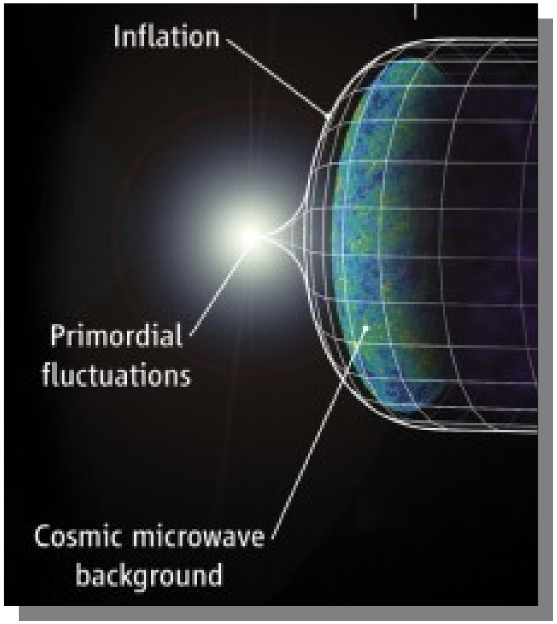
Inflation Recipe

Various potentials (attrition with observation)



Inflation Recipe

$$\phi(x,t) = \bar{\phi}(t) + \delta\phi(x,t)$$



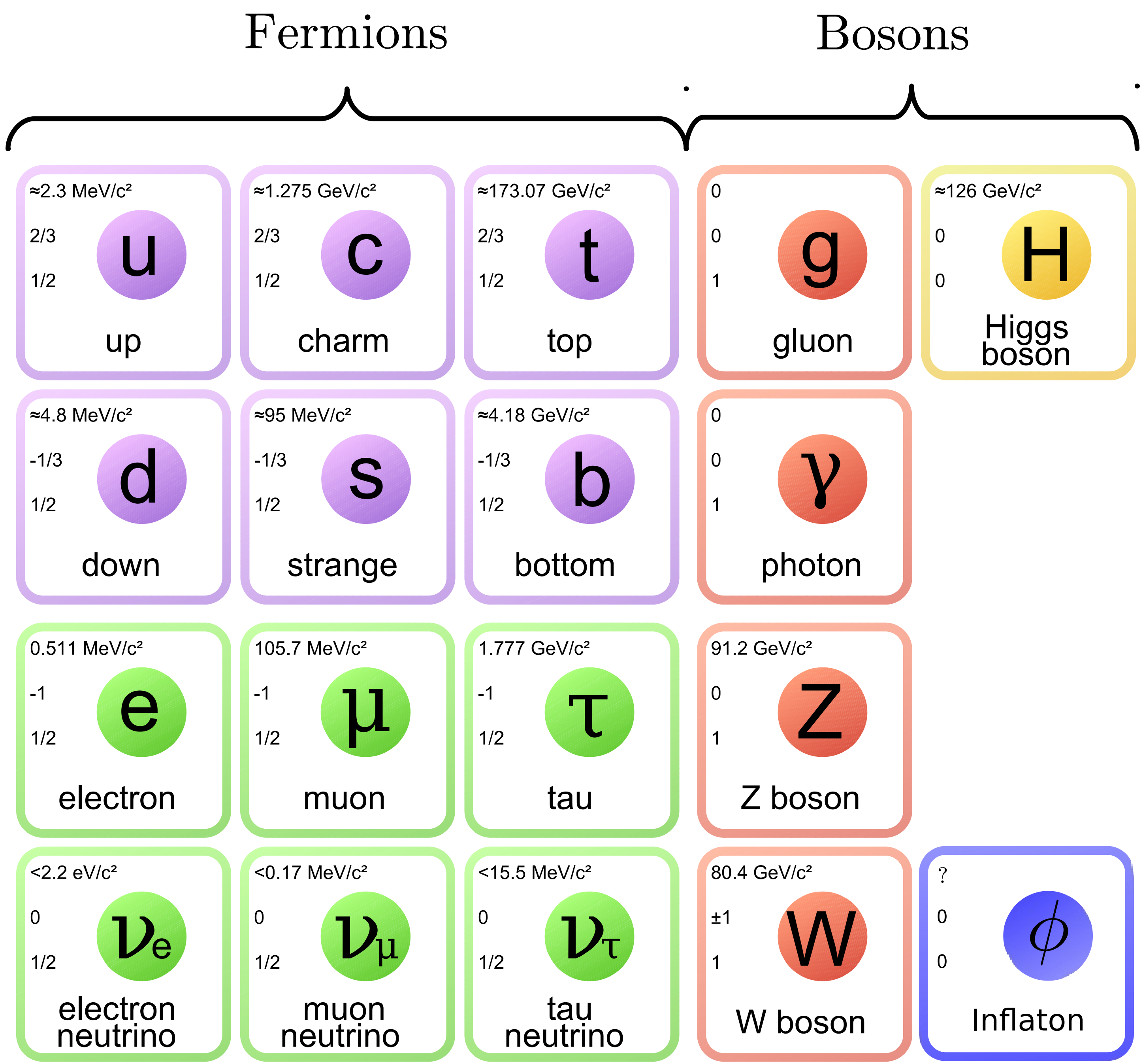
Classical scalar field in FLRW background

Local isotropy and homogeneity (CMB, galaxy count)

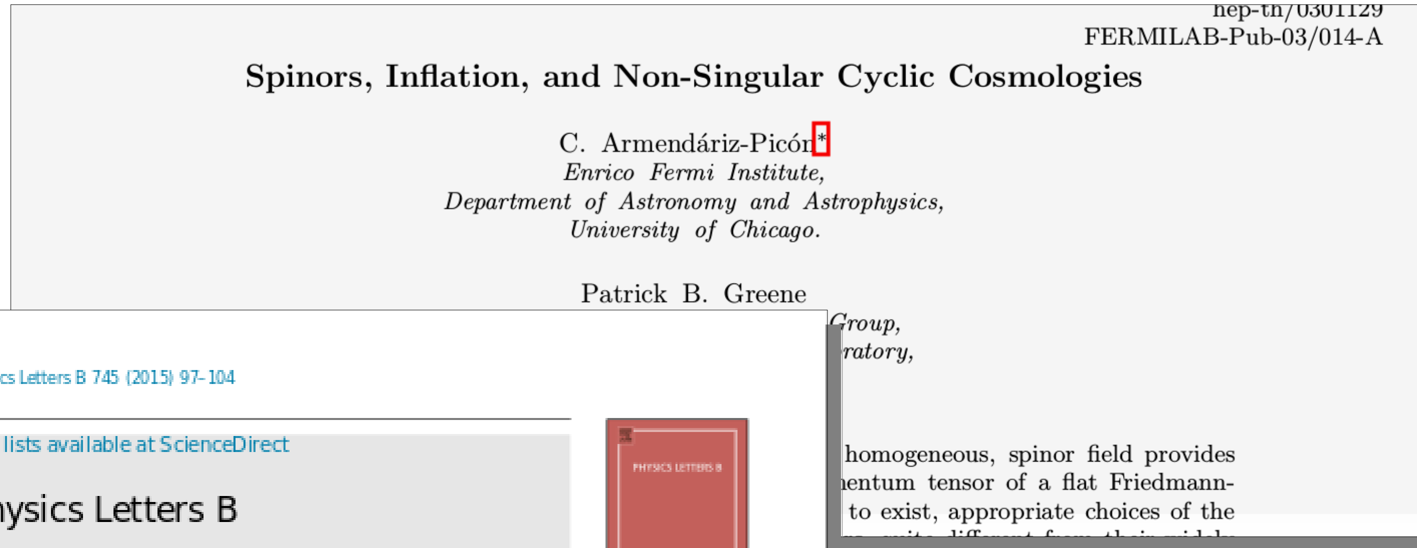
Various potentials (attrition with observation)

Quantized fluctuations over classical background

Inflation and Standard Model



Fermion Inflation



hep-th/0301129
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Spinors, Inflation, and Non-Singular Cyclic Cosmologies

C. Armendáriz-Picón*
Enrico Fermi Institute,
Department of Astronomy and Astrophysics,
University of Chicago.

Patrick B. Greene
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...*ratory,*

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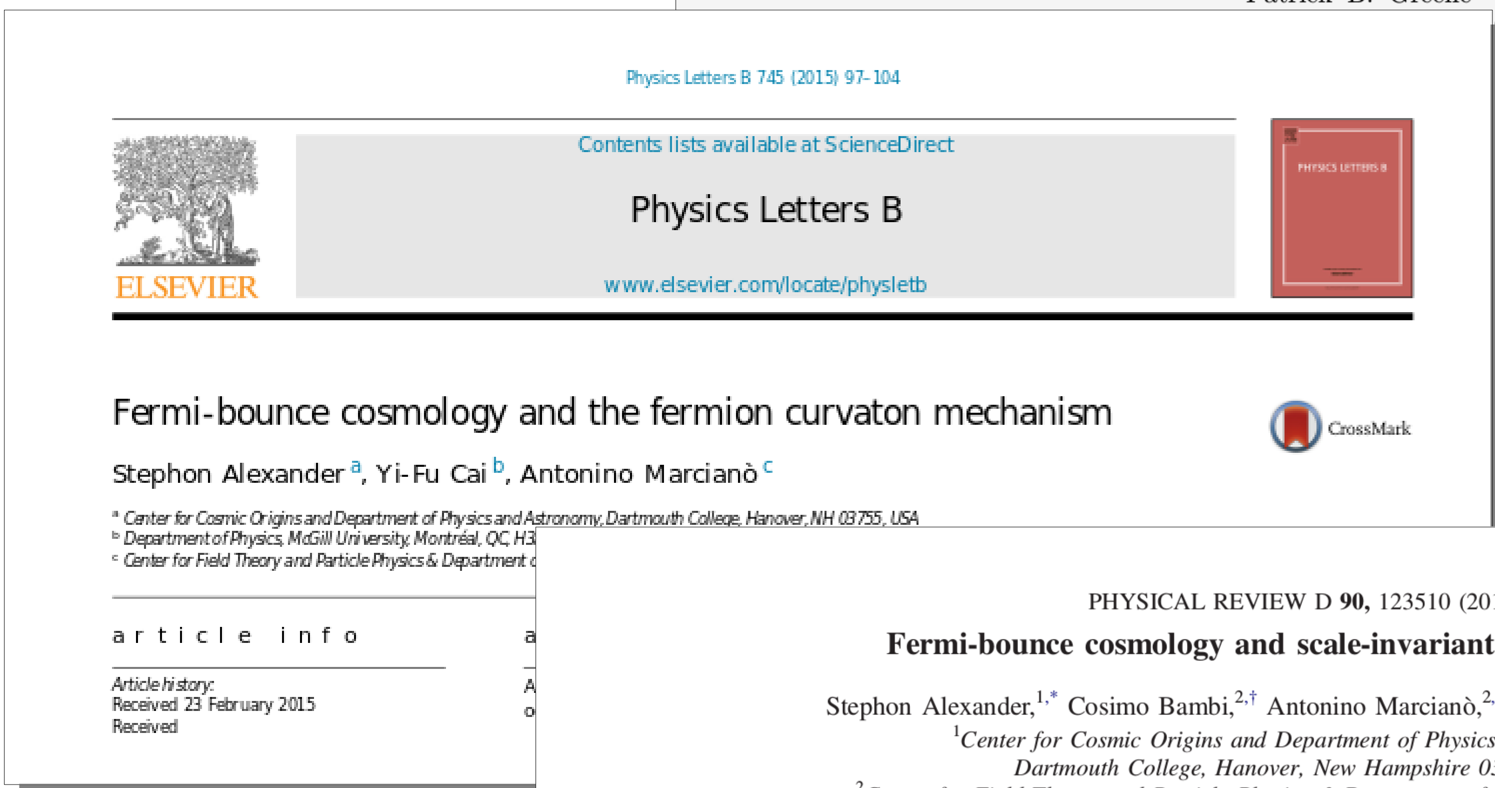

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Fermi-bounce cosmology and the fermion curvaton mechanism

Stephon Alexander [a], Yi-Fu Cai [b], Antonino Marcianò [c]

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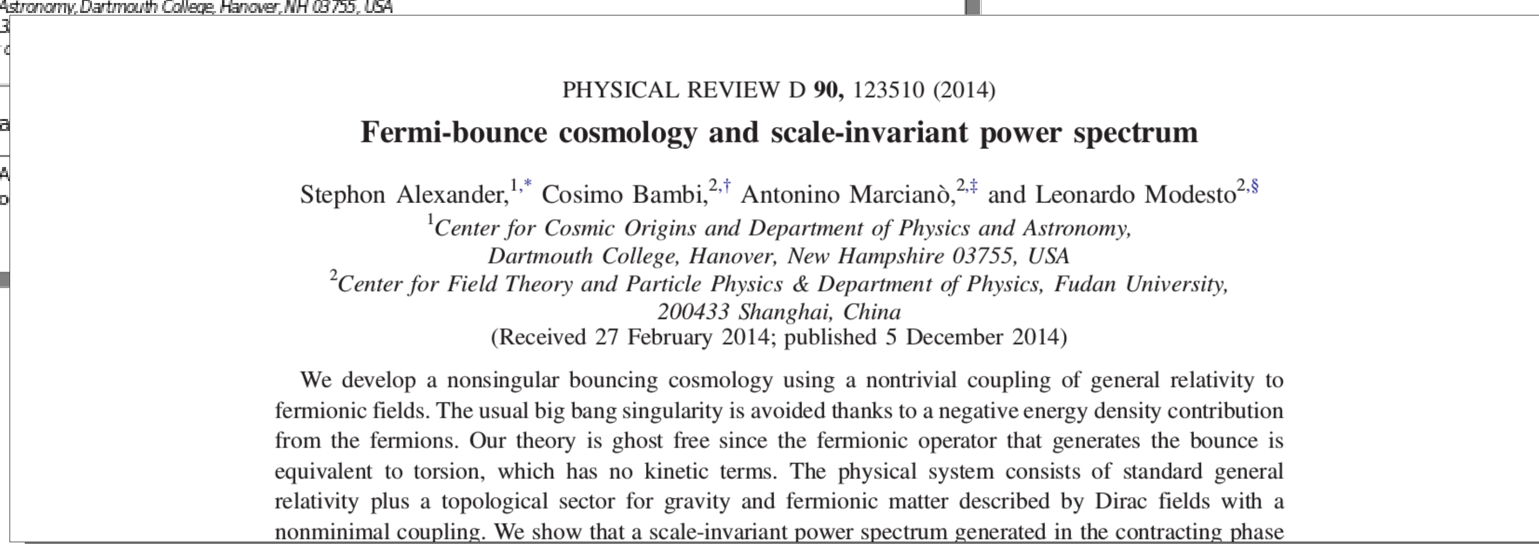

PHYSICAL REVIEW D **90,** 123510 (2014)

Fermi-bounce cosmology and scale-invariant power spectrum

Stephon Alexander,[1,*] Cosimo Bambi,[2,†] Antonino Marcianò,[2,‡] and Leonardo Modesto[2,§]
[1]*Center for Cosmic Origins and Department of Physics and Astronomy, Dartmouth College, Hanover, New Hampshire 03755, USA*
[2]*Center for Field Theory and Particle Physics & Department of Physics, Fudan University, 200433 Shanghai, China*
(Received 27 February 2014; published 5 December 2014)

We develop a nonsingular bouncing cosmology using a nontrivial coupling of general relativity to fermionic fields. The usual big bang singularity is avoided thanks to a negative energy density contribution from the fermions. Our theory is ghost free since the fermionic operator that generates the bounce is equivalent to torsion, which has no kinetic terms. The physical system consists of standard general relativity plus a topological sector for gravity and fermionic matter described by Dirac fields with a nonminimal coupling. We show that a scale-invariant power spectrum generated in the contracting phase

Fermion Inflation

hep-th/0301129
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Spinors, Inflation, and Non-Singular Cyclic Cosmologies

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"What you are doing is all wrong!"
Prof. Misao Sasaki, Taipei, December 2015

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Fermion perturbations

"There are no linear perturbation of fermion fields"
Prof. Misao Sasaki, Taipei, December 2015

Linear perturbations of fermionic bilinears

$$\delta\left(\bar{\psi}M\psi\right) = \delta\bar{\psi}M\psi + \bar{\psi}M\delta\psi$$

Classical Background = EV on a state (FLRW symmetries)

$$\langle\psi\rangle = 0 \rightarrow \delta\left(\bar{\psi}M\psi\right) = 0$$

Fermion perturbations

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Linear perturbations of fermionic bilinears

$$\delta\left(\bar{\psi}M\psi\right) = \delta\bar{\psi}M\psi + \bar{\psi}M\delta\psi$$

Three reasons because this argument is not accurate

1) Dirac fields are pure quantum objects

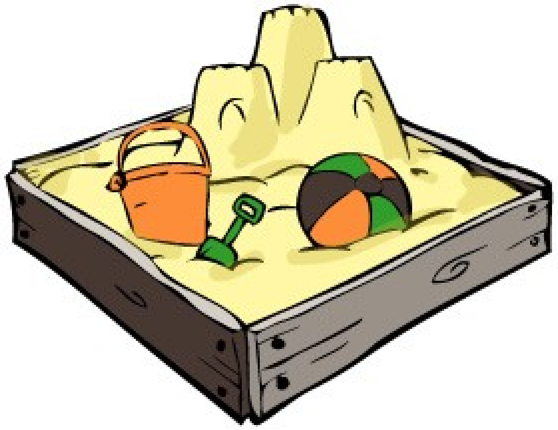
2) Expectation Value of a single fermion field is not observable

3) Separate quantization of background and perturbation fields.

We need to give a generalization of the definition of "perturbation".

Bosonic coherent cosmological perturbation

Scalar Field Inflation



Build a Recipe

PD, Antonino Marciano' - arXiv:1605.09337 (10.1103/PhysRevD.94.123517)

Bosonic coherent cosmological perturbation

Build a Recipe from Scalar Field Inflation

Define the appropriate coherent state

$$|\alpha\rangle = e^{\int_k \alpha(k) a_k^\dagger - \alpha^*(k) a_k} |0\rangle$$

Interpretation: $\langle\alpha| \hat{N} |\alpha\rangle = \int_k |\alpha(k)|^2$

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Bosonic coherent cosmological perturbation

Build a Recipe from Scalar Field Inflation

Define the appropriate coherent state $|\alpha\rangle = e^{\int_k \alpha(k) a_k^\dagger - \alpha^*(k) a_k} |0\rangle$

Classical value of the field = Expectation Value on coherent state

$$\phi_\alpha(x) \equiv \langle\alpha|\,\phi(x)\,|\alpha\rangle = \int_k \left(\alpha(k)e^{-ikx} + \alpha^*(k)e^{+ikx}\right) \, .$$

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Classical perturbation fields = Expectation Value on a perturbed coherent state

$$\langle\alpha + \delta\alpha|\, \phi(x)\, |\alpha + \delta\alpha\rangle = \phi_\alpha(t) + \phi_{\delta\alpha}(x,t)$$

Perturbed quantities in terms of operator on perturbed state

$$\delta G_{\mu\nu} = 8\pi G \langle\alpha + \delta\alpha|\, \widehat{T_{\mu\nu}(\phi)}\, |\alpha + \delta\alpha\rangle\, |_{O(\delta\alpha)}$$

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Perturbed quantities in terms of operator on perturbed state

$$\hat{\zeta}(x) = -\frac{\rho}{3\,\langle\alpha|\rho + p|\alpha\rangle} \qquad P_\zeta = \lim_{y\to x} \langle\alpha + \delta\alpha|\, \hat{\zeta}(x)\hat{\zeta}(y) \,|\alpha + \delta\alpha\rangle \,|_{O(\delta\alpha)^2}$$

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Classical value ... erent state

ϕ_α (... $x)$.

By construction we recover the standard results

Classical perturbation fields = Expectation Value on a perturbed coherent state

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Fermionic coherent cosmological perturbation

Fermionic coherent state? - Too naive

$$|\alpha\rangle = e^{\int d^3k\, \alpha(k) a_k^\dagger - \alpha^*(k) a_k} |0\rangle \longleftarrow \text{Not in Fock Space}$$

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Fermionic coherent cosmological perturbation

Fermionic coherent state? - Too naive

$$|\alpha\rangle = e^{\int d^3k\, \alpha(k) a_k^\dagger - \alpha^*(k) a_k} |0\rangle$$

Generalized SU(2) coherent state (pairs)

$$|\alpha\rangle = e^{\int d^3k\, \alpha(k) a_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger - \alpha^*(k) a_{k\uparrow} b_{-k\downarrow}} |0\rangle \longleftrightarrow |\vec{n}\rangle$$

$$J_1 = \frac{1}{2}\left(a^\dagger b^\dagger + h.c.\right)$$

$$J_2 = -\frac{i}{2}\left(a^\dagger b^\dagger - h.c.\right)$$

$$J_3 = \frac{1}{2}\left(a^\dagger a + b^\dagger b - 1\right)$$

Density of pairs

$$\langle \vec{n}| \bar{\psi}\psi |\vec{n}\rangle = \langle \vec{n}| \int_k \vec{\xi} \cdot \vec{J}_k |\vec{n}\rangle = \int_k \vec{\xi} \cdot \vec{n}_k$$

Infinitesimal rotation around a finite one

$|\vec{n} + \delta\vec{n}\rangle$

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Classical perturbations:

$$\delta G_{\mu\nu} = 8\pi G \langle \vec{n} + \delta\vec{n} | \widehat{T_{\mu\nu}(\bar{\psi}, \psi)} | \vec{n} + \delta\vec{n} \rangle |_{O(\delta\vec{n})}$$

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Compute relevant cosmological quantities:

$$P_\zeta = \lim_{y \to x} \langle \alpha + \delta\alpha | \hat{\zeta}(x) \hat{\zeta}(y) | \alpha + \delta\alpha \rangle |_{O(\delta\alpha)^2}$$

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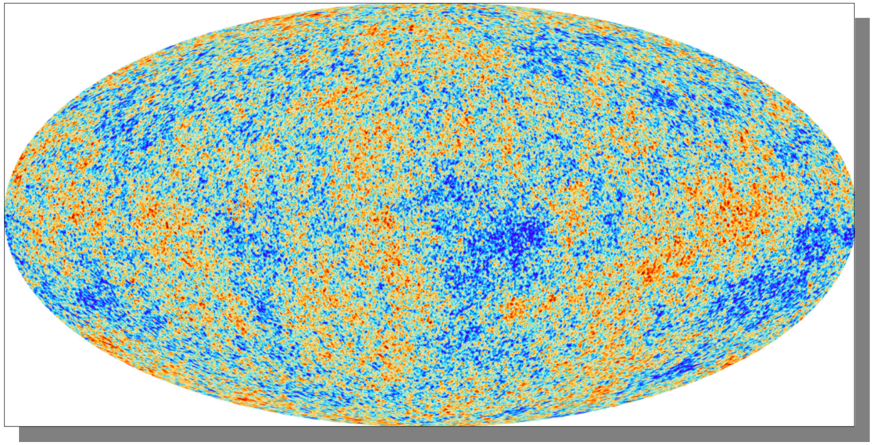
Non Bunch Davies group coherent states

Inflation initial state?

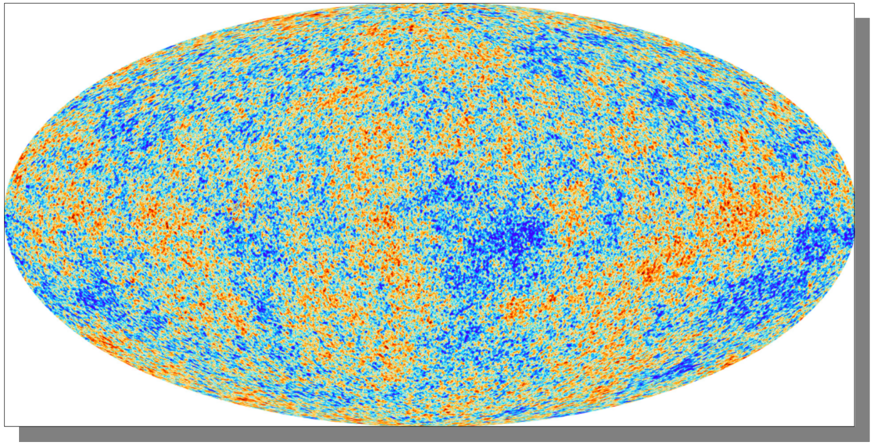


Heavily rely on the selected quantum state

- estimate of the non-Gaussianities,
- power spectra of CMBR,
- tensor to scalar ratio parameter.

Non Bunch Davies group coherent states

Inflation initial state?



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Bunch Davies vacuum (dS spacetime isometries, approach Minkowski vacuum in deep UV, time-independent, stationary state)

Generalization to Non-Bunch Davies vacuum (misleading name, excited initial state for inflation, most general solution of the mode equation, parametrize our ignorance on the pre-inflationary era)

Non Bunch Davies group coherent states

Inflation initial state?

Generalization to Non-Bunch Davies vacuum (parametrize our ignorance on the pre-inflationary era)

$$u_k^{NBD}(\eta) = \alpha_k u_k^{BD}(\eta) + \beta_k u_k^{*BD}(\eta) \qquad |\alpha_k|^2 - |\beta_k|^2 = 1$$

Equivalent to:

$$\tilde{a}^\dagger_{\vec{k}} = \alpha_k a^\dagger_{\vec{k}} + \beta_k a_{-\vec{k}}$$

$$|0_{NBD}\rangle = \exp\left[\int_k \gamma(|\vec{k}|) a^\dagger_{\vec{k}} a^\dagger_{-\vec{k}} - \gamma^*(|\vec{k}|) a_{\vec{k}} a_{-\vec{k}}\right] |0_{BD}\rangle$$

$$\alpha_k = \cosh\left(\left|\gamma(|\vec{k}|)\right|\right) \qquad \beta_k = \frac{\gamma(|\vec{k}|)}{\left|\gamma(|\vec{k}|)\right|} \sinh\left(\left|\gamma(|\vec{k}|)\right|\right)$$

Suddhasattwa Brahma, PD, Antonino Marciano' - arXiv:1612.00760

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Exactly an SU(1,1) coherent state:

$$K_+ = a^\dagger_{\vec{k}} a^\dagger_{-\vec{k}}$$

$$K_- = a_{\vec{k}} a_{-\vec{k}}$$

$$K_3 = \frac{1}{2}\left(a^\dagger_{\vec{k}} a_{\vec{k}} + a^\dagger_{-\vec{k}} a_{-\vec{k}} + 1\right)$$

Most general (group theoretical) coherent state that is homogeneous and isotropic

Suddhasattwa Brahma, PD, Antonino Marciano' - arXiv:1612.00760

Non Bunch Davies group coherent states

Probe the SU(N,M) coherent state case, the generators in a Schwinger-like representation are:

$$-a^\dagger_{k_i} a_{k_{i'}} \quad \text{for} \quad i', i = 1, \ldots, n \qquad\qquad a^\dagger_{k_i} a^\dagger_{q_j} \quad \text{for} \quad \begin{cases} i = 1, \ldots, n \\ j = 1, \ldots, m \end{cases}$$

$$a_{q_j} a^\dagger_{q_{j'}} \quad \text{for} \quad j', j = 1, \ldots, m \qquad\qquad a_{k_i} a_{q_j} \quad \text{for} \quad \begin{cases} i = 1, \ldots, n \\ j = 1, \ldots, m \end{cases}$$

Most general form of a coherent state up to irrelevant phases

$$\exp\left[\int_{k_i, q_j} \alpha(k_i, q_j) a^\dagger_{k_i} a^\dagger_{q_j} - \alpha^*(k_i, q_j) a_{k_i} a_{q_j}\right] |0\rangle$$

Invariance under translations fix $k_i = q_j$

Invariance under rotations fix $\alpha(k_i) = \alpha(|k_i|)$

$$\exp\left[\sum_{k_i} \int_{k_i} \alpha(|k_i|) a^\dagger_{k_i} a^\dagger_{-k_i} - \alpha^*(|k_i|) a_{k_i} a_{-k_i}\right] |0\rangle$$

Suddhasattwa Brahma, PD, Antonino Marciano' - arXiv:1612.00760

Conclusions

Introduced a new framework for cosmological perturbation based on coherent states.

- Reproduce the literature
- Generalization to non scalar species
- Fermion fields can indeed contribute to linear cosmological perturbation

Non Bunch-Davies vacuum (initial state for single scalar field inflation) can be interpreted as a SU(1,1) coherent state

- Generalization to other groups is not possible (not compatible with isotropy and homogeneity)