# Computing Lorentzian Spin Foam Amplitudes: *Overview*

Pietro Dona'

Strenuous effort of S. Speziale gang
(P.D. M. Fanizza, G. Sarno, P. Martin-Dussaud)

Loops 2017
Warsaw, Poland
7th July 2017


# Questions?

Is it time for actual calculations?

black hole, cosmology, modified dispersion relations

[M. Christodoulou – Mon] [G. V. Stagno – Fri 5pm]

How well we understand the semi-classical limit?

check of existing analysis, meaning of large spin, extended

[J. Barret et al. – '09-'10]

What's the semi-classical limit of KKL model?

generalized EPRL, general valency SU(2) BF [B. Bahr et. al '16-'17]

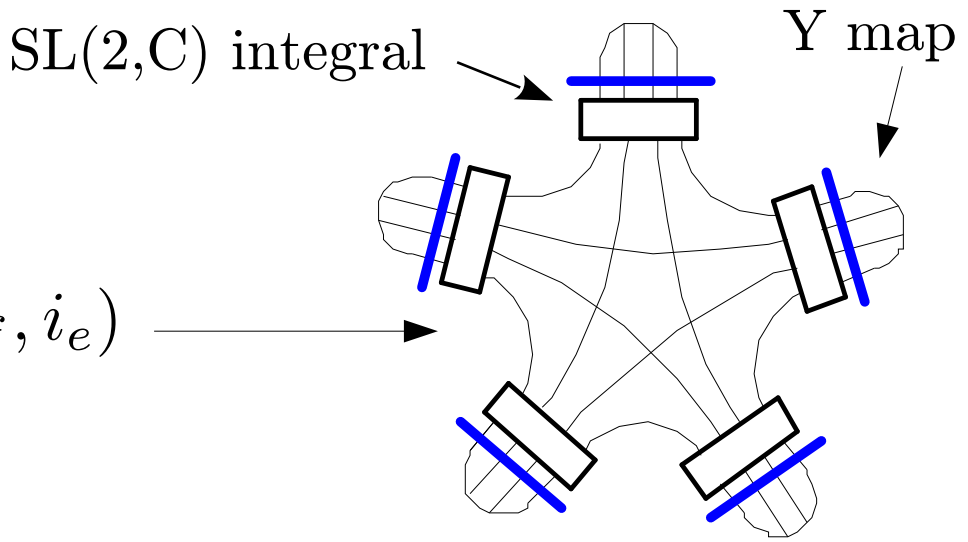
Is it necessary to renormalize Spin Foam?

divergences, various models

[B. Bahr group; D. Oriti group: M. Finocchiaro talk;
B. Dittrich program] [A. Riello '13]
[E. Livine, V. Bonzom and may others]

# Decomposition of the Amplitude

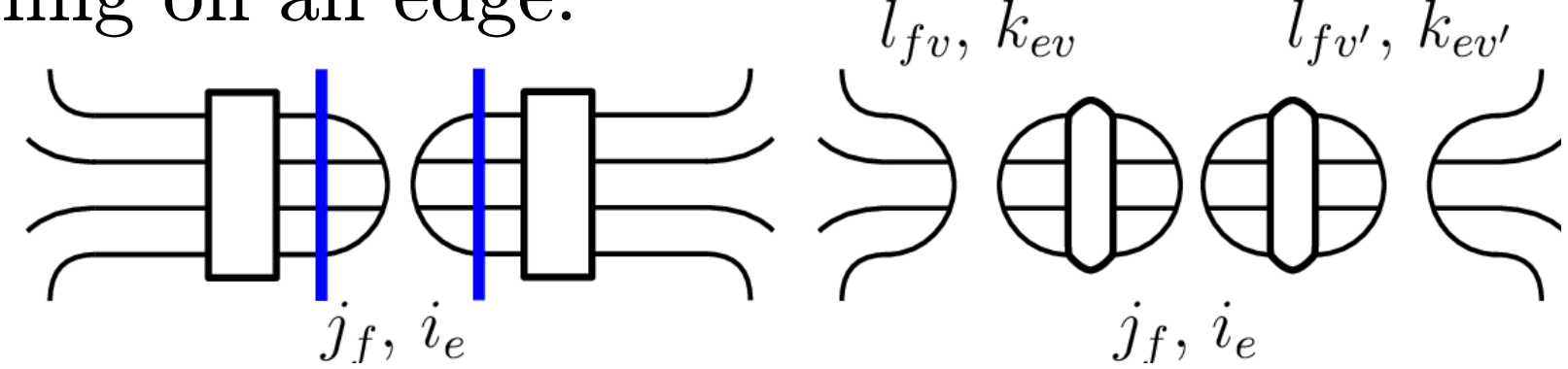
[S. Speziale '16]




EPRL amplitude:

$$Z_{\mathcal{C}} = \sum_{j_f, i_e} \prod_f (2j_f + 1) \prod_v A_v (j_f, i_e)$$

Zooming on an edge:



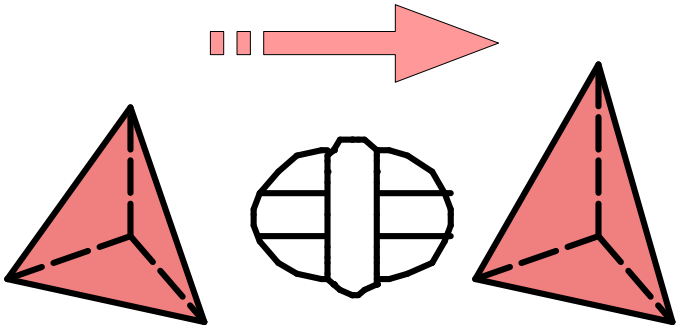

Booster $B_4 (j_f, i_e, l_{fv}, k_{ev}; \gamma)$

Take at home message

We decomposed the EPRL amplitude into a superposition of SU(2) ones weighted by the Boosters (one per half-edge)

# Properties of the Booster

4 valent



Relatively simple expression
one dimensional
well localized integral (boost)

Sum over boost connecting two quantum tetrahedra
intriguing semi-classical limit
[wip see the talk of Pierre Martin-Dussaud – Today 6pm]

Select only particular configurations
direction of the boost, isosceles configuration

Simplified Model

$$\sum_{l_{fv}} \to l_{fv} = j_f$$

Simplifications in the diagonal case [Puchta '13]
peaked on the trivial boost
numerical computation is much faster
capture good features in some regime

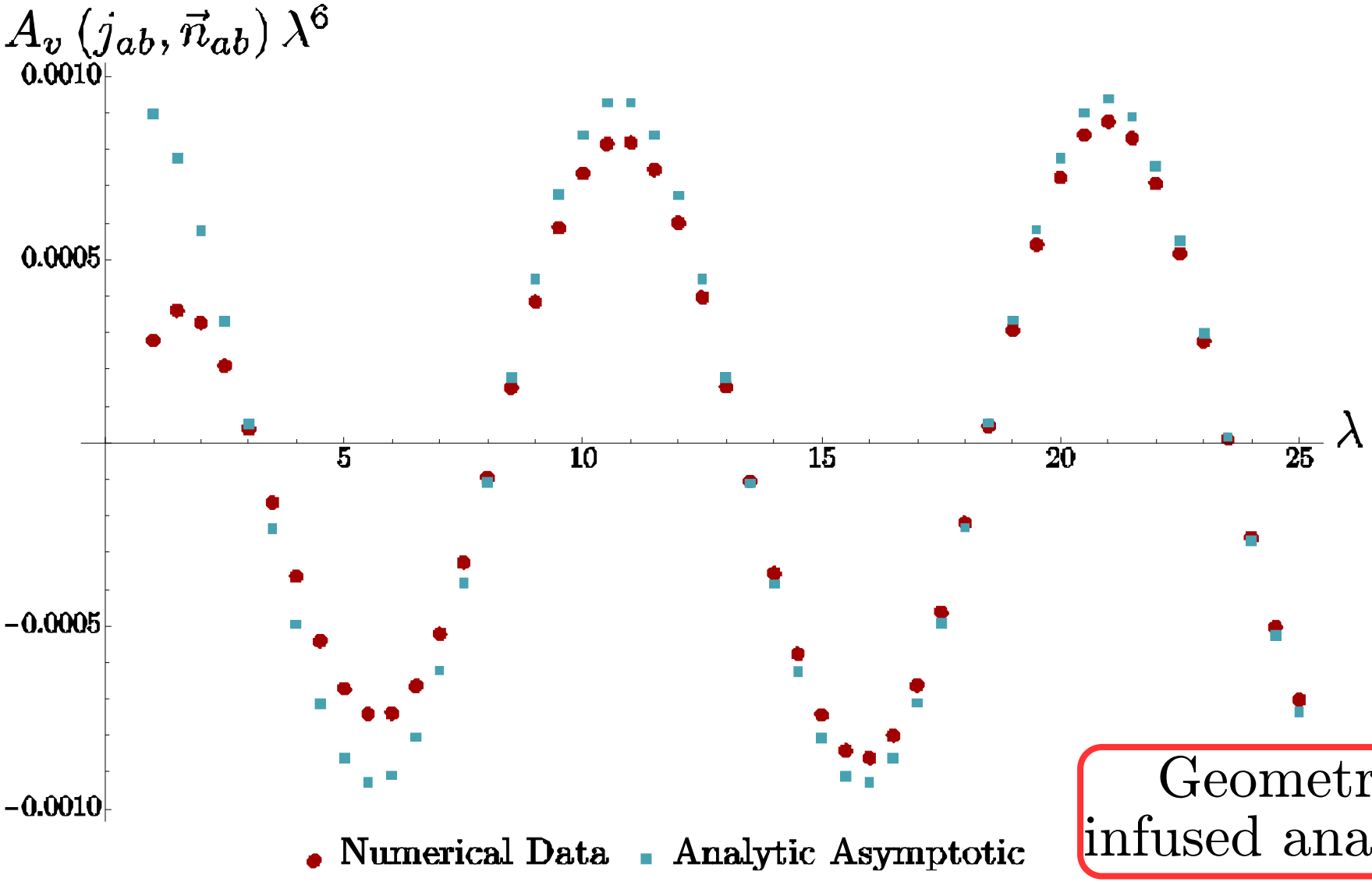
# Numerics & Asymptotic - SU(2)

[M. Fanizza Talk Mon]

First numerical confirmation of the asymptotic formula

power law decay, frequency (Regge action), relative phase, access the semi-classical region at low spins

[J. Barrett et al. – '09-'10]




Geometry infused analysis

# Numerics & Asymptotic - EPRL

[G. Sarno Talk Tue]

Two new layer of complication respect to SU(2)

efficient computation of the Boosters

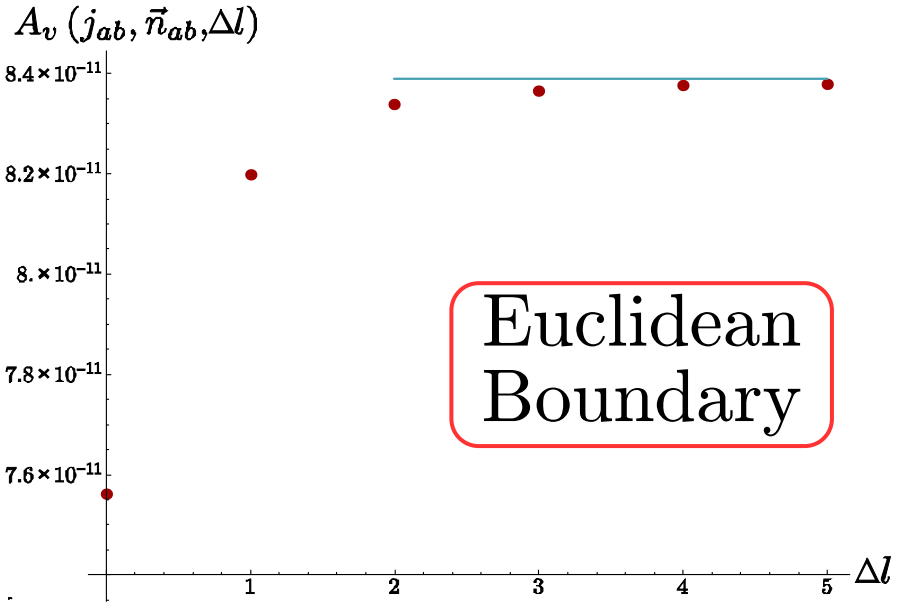
infinite summation of configurations

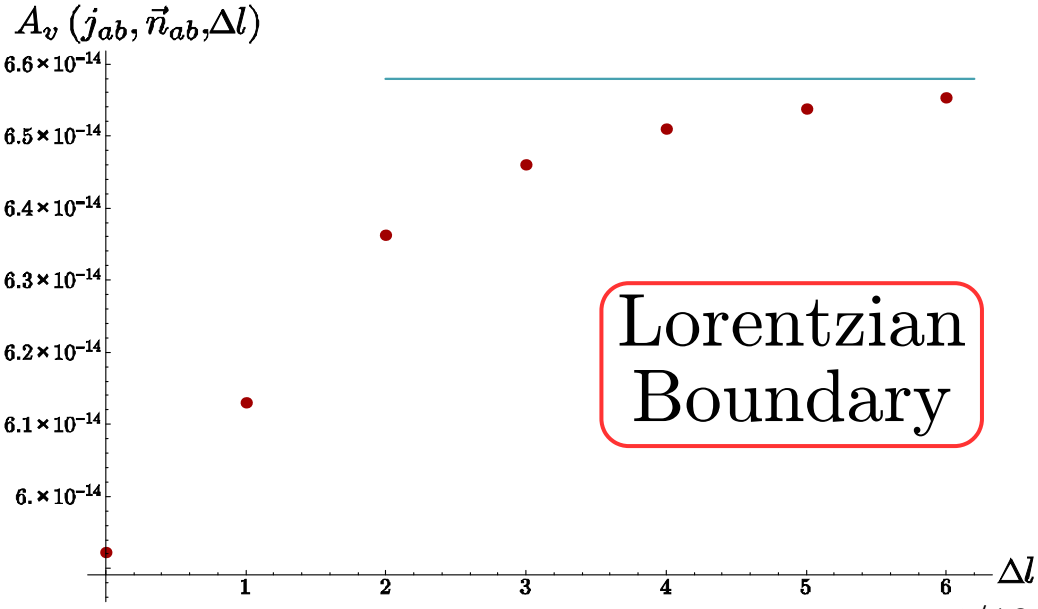
Fast convergence

comeback of the simplified model

shell-like ordering of the contributions

needs for a lot of optimization



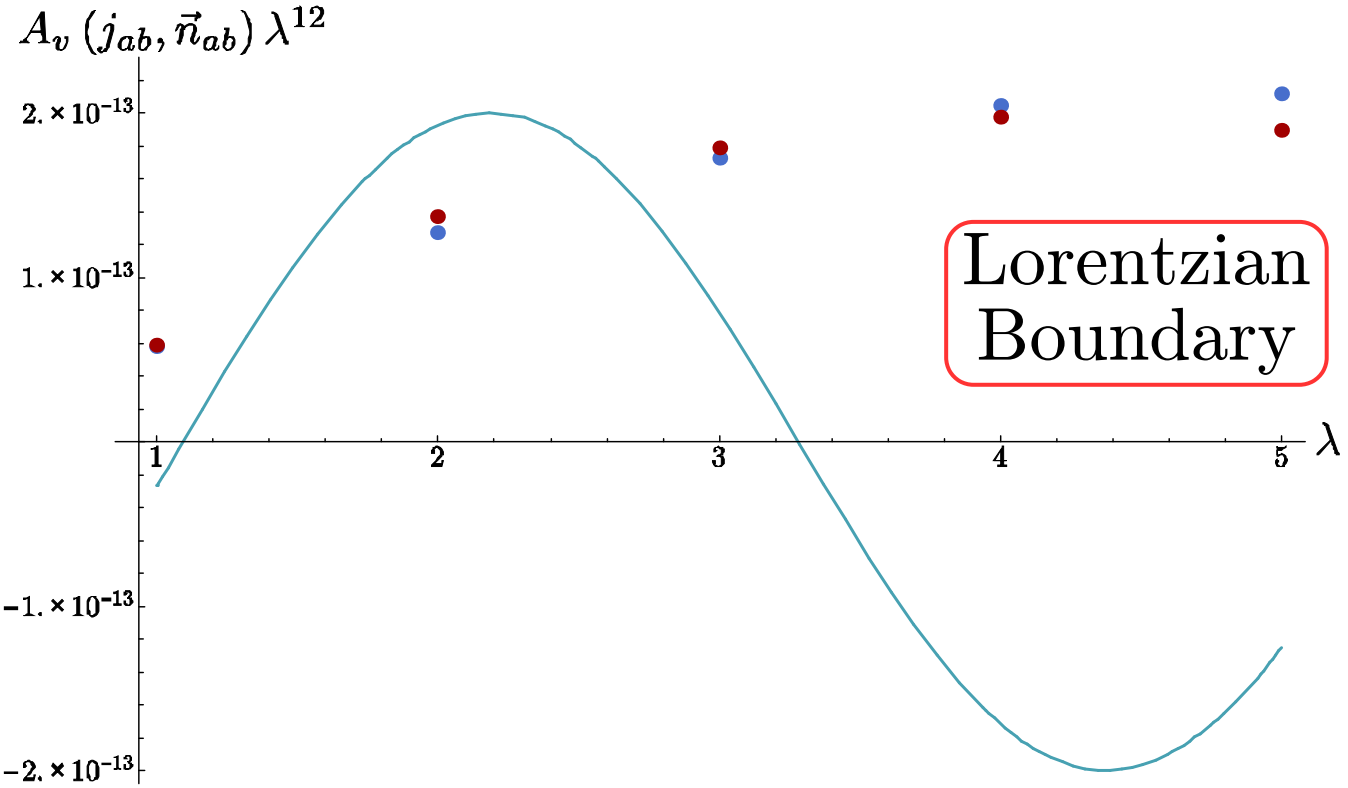
# Numerics & Asymptotic - EPRL

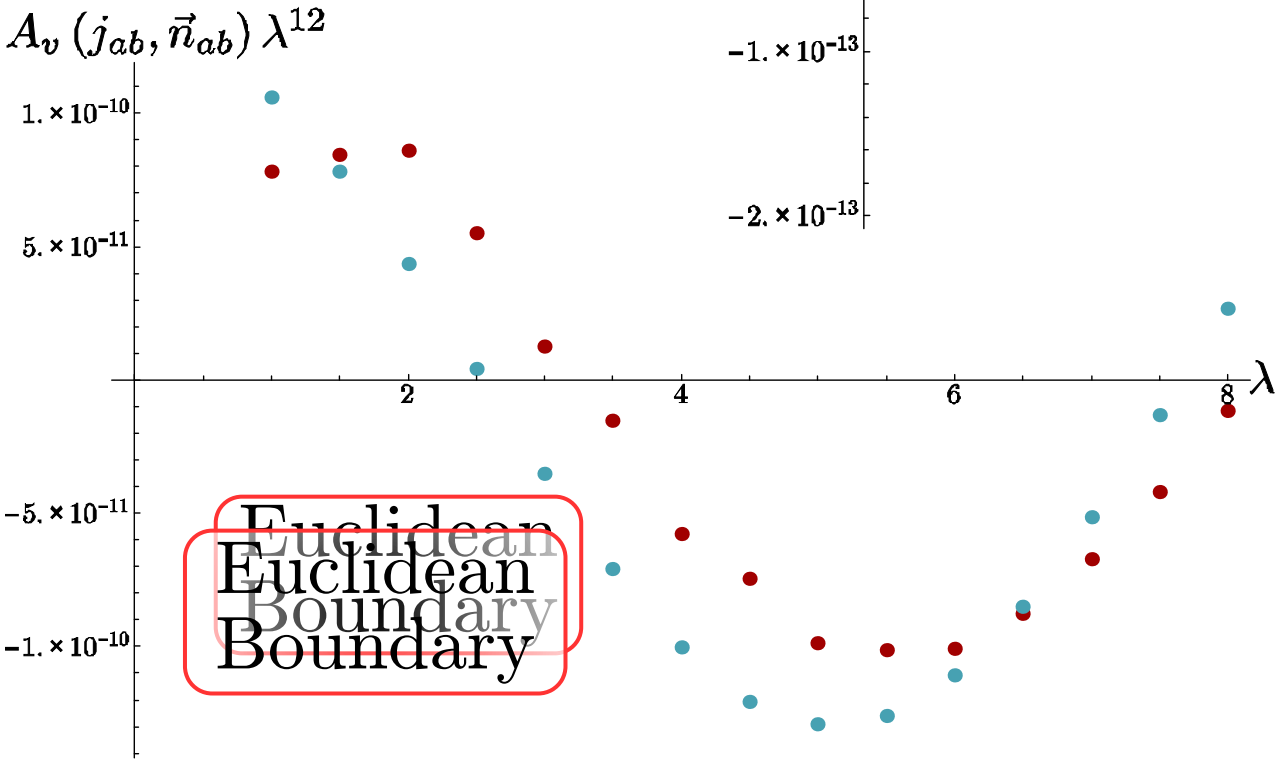
## Asymptotic behavior of both the amplitudes

we are far away from semi-classical region

exploration of pure quantum region

possible selection rule




simplified model!

phase mismatch

# Numerics & Asymptotic - EPRL

[G. Sarno – Tue]

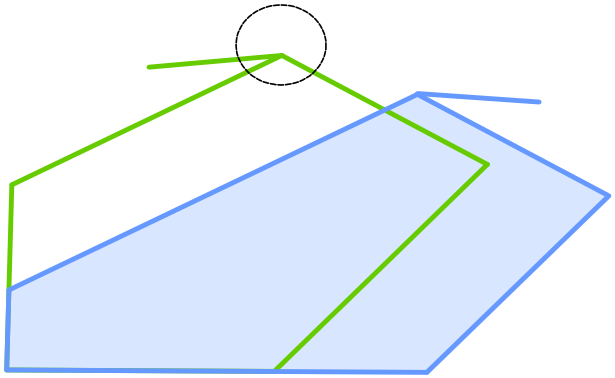
# Asymptotic analysis general valency vertex SU(2)

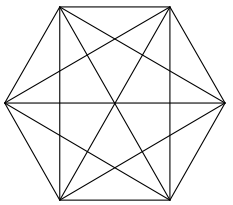
The amplitude generalization is straightforward

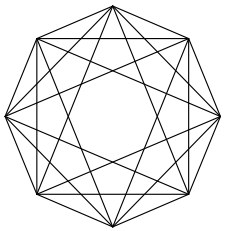
the saddle point analysis generalize smoothly [B. Bahr et al.]

Taxonomy of options

| Dofs | Geometry type | Saddle points |
|---|---|---|
| 5L-6N | twisted | 0 |
| 3L-3N | vector *(anti-parallel)* | 1 |
| ? | conformal twisted *(angle-matching)* | 2 |
| E | Regge *(shape-matching)* | 2 |
| 4N-10 | Polytope *(flat embedding)* | 2 |



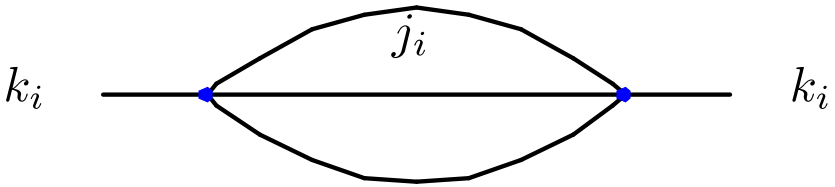




# Divergences

Self energy diagram is Log divergent (EPRL) [Riello '13]
utilized the asymptotic formula
other graphs?

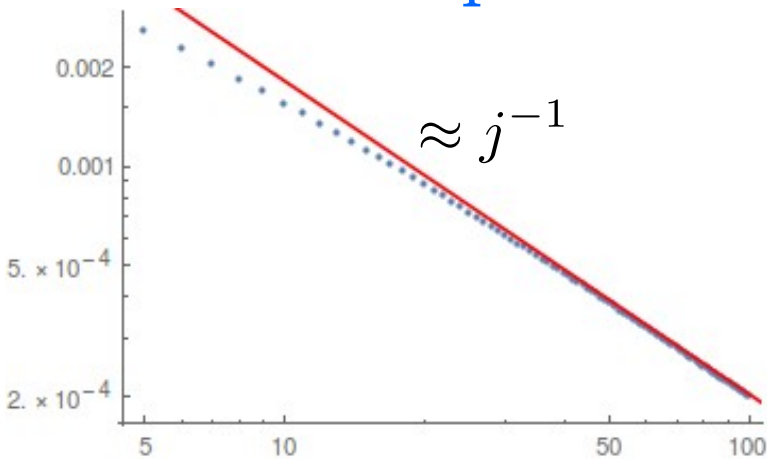
It is possible to find a more systematic procedure?
e.g. SU(2) BF theory in 3D

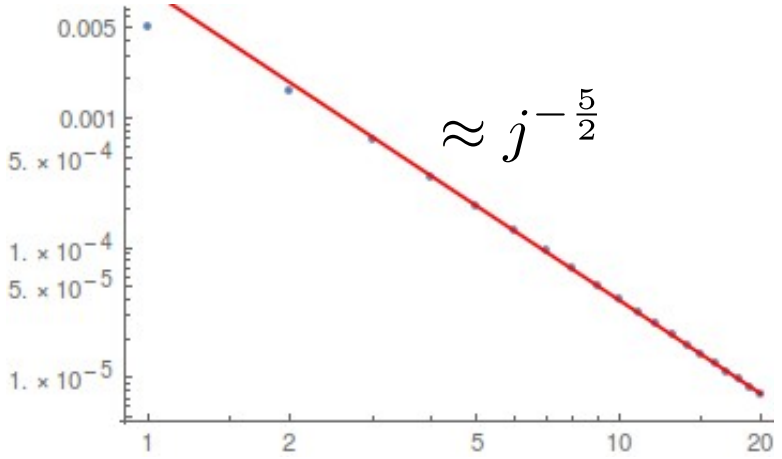



One triangular inequality per edge

$$\sum_{j_1 j_2 j_3}^{\Lambda} d_{j_1}^{\mu} d_{j_2}^{\mu} d_{j_3}^{\mu} \left\{ \begin{array}{ccc} k_1 & k_2 & k_3 \\ j_1 & j_2 & j_3 \end{array} \right\}^2 \approx \sum_{\lambda}^{\Lambda} \lambda^{3\mu} \frac{1}{\lambda} \approx \Lambda^{3\mu}$$

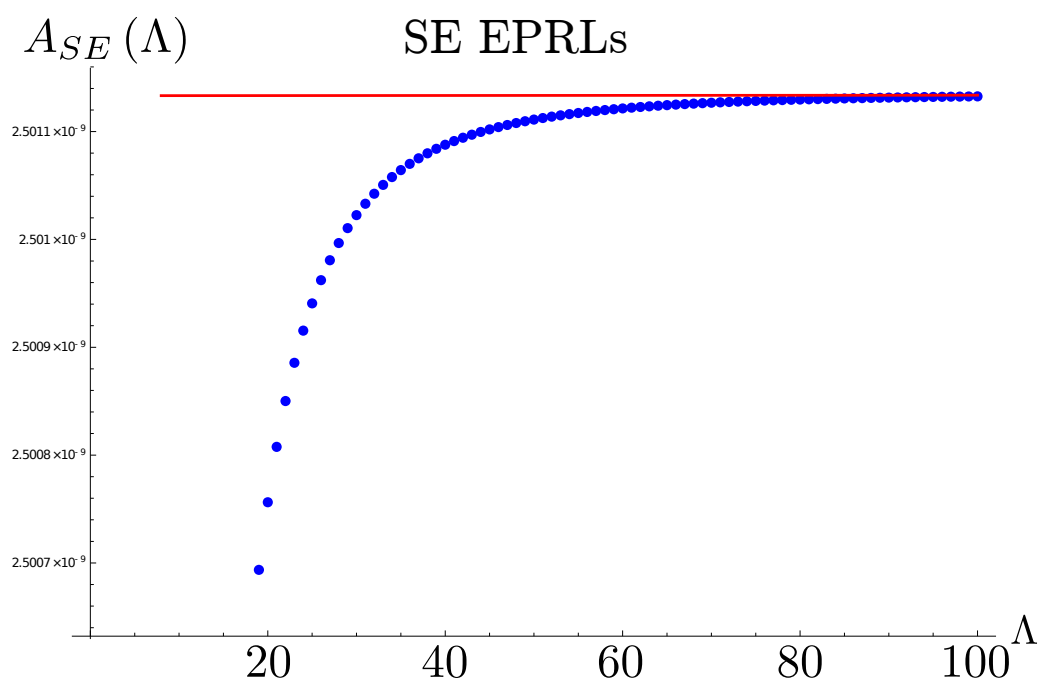
Non homogeneous scaling of the Booster
numerical and simplified model

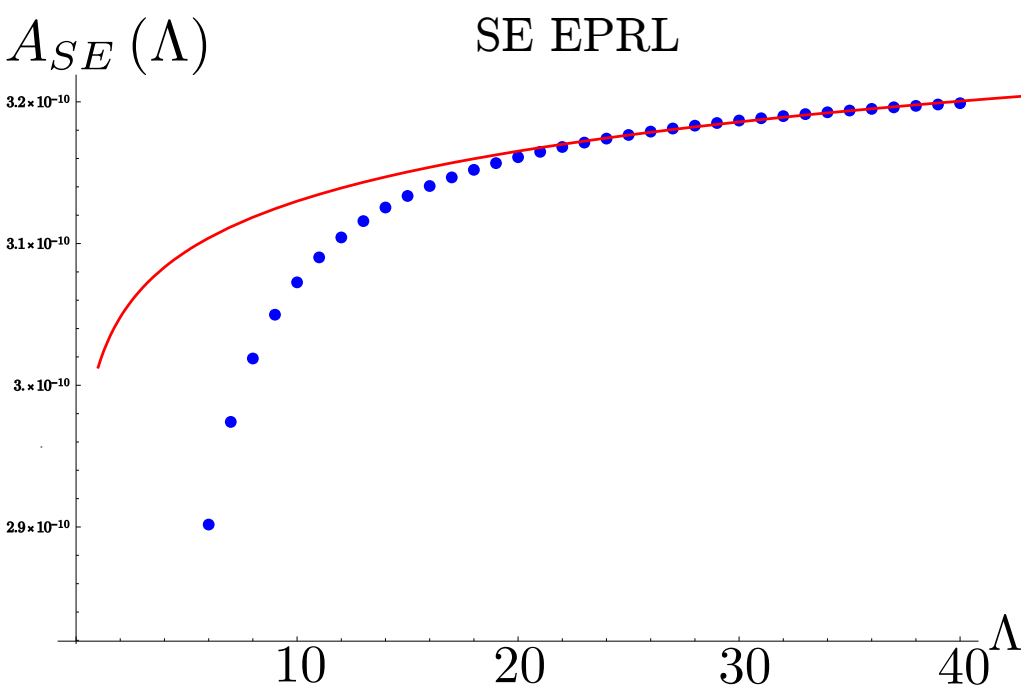


# Divergences

| | 3D | | 4D | |
|---|---|---|---|---|
| | Self Energy | 14 Move | Self Energy | 15 Move |
| BF | $\Lambda^{3\mu}$ | $\Lambda^{4\mu-1}$ | $\Lambda^{3(2\mu+1)}$ | $\Lambda^{5(2\mu+1/2)}$ |
| EPRLs | $\Lambda^{3(\mu-2)}$ | $\Lambda^{4(\mu-3)-1}$ | $\Lambda^{6\mu+3-4\cdot5}$ | $\Lambda^{5(2\mu+1/2-5)}$ |
| EPRL | $\log\Lambda$ | Convergent | $*\log\Lambda$ | |



# Answers!

Is it time for actual calculations!
numerical access to spin-foam cosmology

[G. Sarno, S. Speziale, G. V. Stagno, to appear soon– talk today 5pm]

We understand quite well the semi-classical limit!
Numerical confirmation of Barrett analysis,
more geometrical infused,
large spin can be quite small,
hints on the flatness problem

[PD, M. Fanizza, G. Sarno, S. Speziale hopefully very soon]

Not yet at the semi-classical limit of KKL model
generalized EPRL, general valency SU(2) BF

[PD, M. Fanizza, G. Sarno, S. Speziale hopefully very soon]

Back of the envelope estimation of divergencies
warming up with simpler models

[PD, S. Speziale, work in progress]

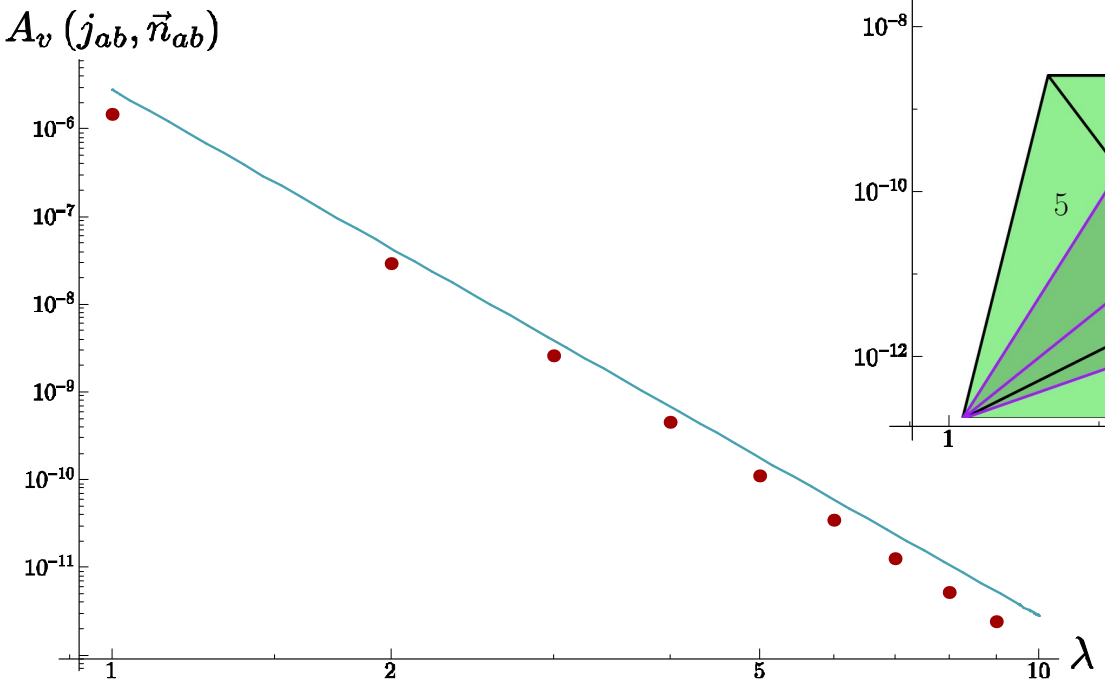
Thank you for your attention!

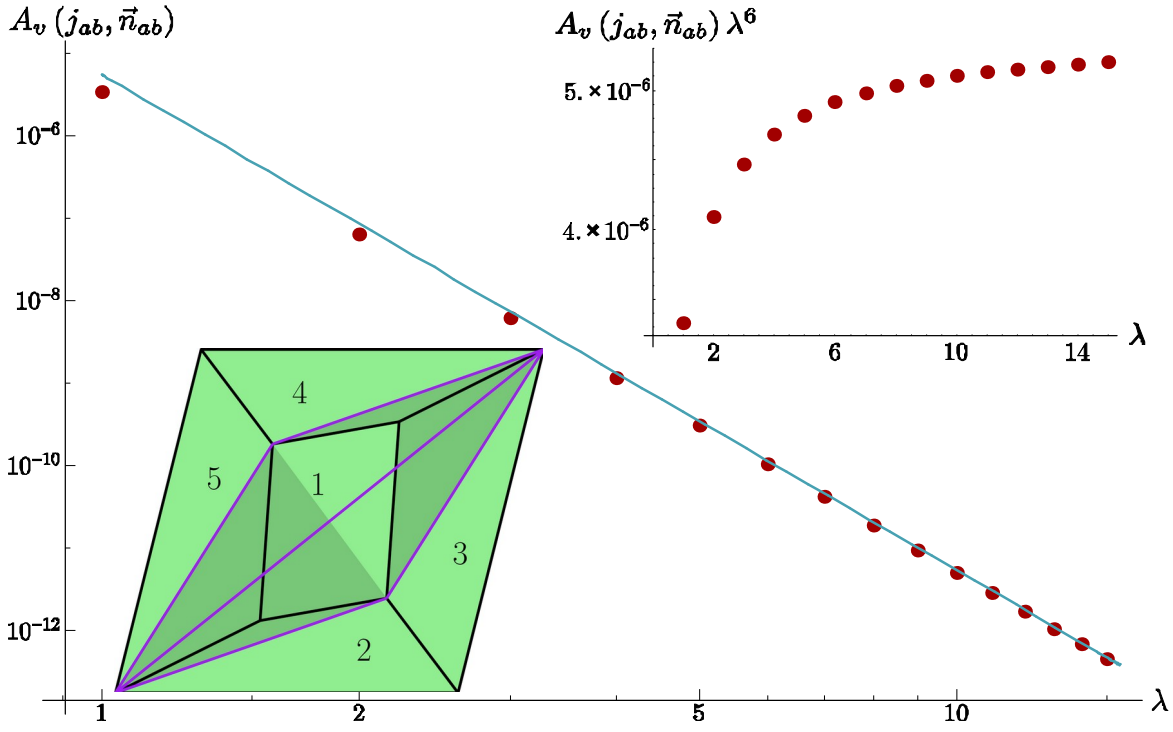
# Numerics & Asymptotic - SU(2)

[M. Fanizza – Mon]

Vector geometries test and characterization

power law scaling, no oscillations, 5 dimensional manifold







Generic Twisted geometry

exponential suppression, e.g. Lorentzian 4-simplex

continue...